Parallel Transforming Percentage (PTP) Theorem

Abstract

The parallel Transforming Percentage theorem is introduced to solve some mysteries of geometry problems and help to build magnified objects or products easier in 2D or 3D parallel transforming geometry for lines, shapes and objects.

The theorems:

1. *All spheres are parallel, and they are only different in form of percentage.*
2. *All circles are parallel on the same surface or on other parallel surfaces, and they are only different in form of percentage.*
3. *All lines, shapes, and objects are zoomified, are parallel transforming from its original, and they are only different in form of percentage.*
4. *A line that parallels with a side of a triangle will divide the other 2 sides into the same percentage on both sides as well as its parallel side.*
5. *A line that parallels with the bases of a trapezoid will divide the 2 sides into the same percentage on both sides, and this line is equal to the small base plus the parallel percentage of the difference of the 2 bases.*
6. *Ellipse is a closed curve with the perimeter that divides all the chord lines of the outer circle with radius **R** that parallel with the fixed short radius **r** of the ellipse into the same percentage on each chord line for all the chord lines with the same percentage of* $\frac{r}{R}$.
7. *The e-chords are the **n** lines adjacent to the **n** equal angles in a circle where **n>2**, are equal to **2** times of the circle radius multiply by **sine** of **π/n**.*
8. *The e-chord shape with all **n** sides equal is the largest shape compares to other shapes with the same number of **n** chord lines that are not equal in a circle, and the shape with more e-chord lines has larger area.*
9. *The rectangle in an ellipse which belongs to the e-chord square of the outer circle of the ellipse, has the largest area with the width equals to* $\sqrt{2}$ *multiply by the fixed long radius **R,** and the height equals to* $\sqrt{2}$ *multiply by the fixed short radius **r** of the ellipse.*

Parallel Transforming Percentage (PTP) Theorem

I. Introduction

There are 3 types of transforming geometry; line parallel transforming, surface area parallel transforming and object zoomification transforming. Line parallel transforming will be introduced in triangle and trapezoid. Surface parallel transforming will be also introduced in trapezoid in zoomification transforming. Object zoomification transforming will be introduced in spheres and blocks. These parallel transforming can be used to calculate sides, area and volume faster, and can also be used to pin point the surface center and object center. This PTP theorem will be applied in real life section to find the surface center, object center, and redefine the current ellipse definition. Parallel Transforming can be used to find object size or planet size while travelling in outer space where Doppler radar could not able to use.

The following notations and definitions of words and symbols are using in this document.

1. Key ratio definition: K_r is the key ratio of a new value with the original value. For instance, $K_r = a'/a = b'/b$;
2. Percentage Function definition: P%(x) is percentage function of x without specific value of percentage. For instance, $K_r = a'/a = b'/b$; then $P\%(a) = a' = aK_r$; and $P\%(b) = b' = bK_r$;
3. Percentage expression with a value: P(n%x) is the percentage function of n% value of x where K_r = n%. For instance, x = 123 and K_r = 25% = 0.25; then P(n%x) = P(25%123) = (0.25) 123 = 30.75;
4. Percentage complex expression: P(n%(x + y + z)) is the percentage function of n% value of expression (x + y + z).
5. Object Center is the mass center of an object, and surface center is the mass center of a shape or a surface with assumption that objects and shapes with the same material and density and the shapes with the same thickness.

6. Apply PTP theorem to correct the existing definition of ellipse and comes up with new definition, plus recommendation of using fixed short radius r and fixed long radius R instead of 'a' and 'b' for the width and the height of ellipse. With this recommendation, the younger generations will learn the ellipse and circle better. Ellipse is a circle rotates in certain angle on its plane; where R is the radius of the circle and small r is the radius of the circle lying on the vertical plane when looking from the front view, which can be called small radius 'r' and radius of the circle can be called big radius 'R'.

II. Proofs of Theorems

1. Triangle Parallel Transforming Percentage

Theorem-4: *A line that parallels with a side of a triangle will divide the other 2 sides into the same percentage on both sides as well as its parallel side.*

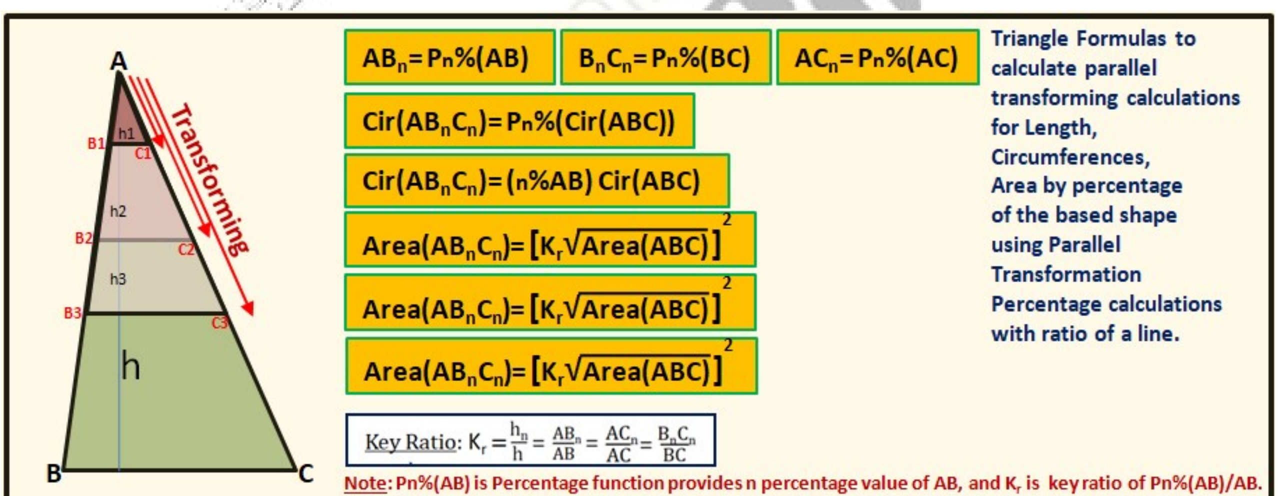

Figure-2.1: Triangle Parallel Transforming in Percentage

Triangle PTP Theorem Proof:

From Figure-2.2 we have,

$$Cos(C) = \frac{a2}{b} = \frac{a2'}{b'};\ \text{Then,}\ \frac{b'}{b} = \frac{a2'}{a2} \rightarrow a2' = a2\frac{b'}{b}$$

$$Cos(B) = \frac{a1}{c} = \frac{a1'}{c'};\ \text{Then,}\ \frac{c'}{c} = \frac{a1'}{a1} \rightarrow a1' = a1\frac{c'}{c}$$

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Parallel Transforming Percentage (PTP) Theorem

$Cos(A1) = \frac{h}{c} = \frac{h'}{c'}$; Then, $\frac{c'}{c} = \frac{h'}{h}$

$Cos(A2) = \frac{h}{b} = \frac{h'}{b'}$; Then, $\frac{b'}{b} = \frac{h'}{h}$

Then, $\frac{b'}{b} = \frac{h'}{h} = \frac{c'}{c} = \frac{a1'}{a1} = \frac{a2'}{a2}$; → $a1 = a1'\frac{c}{c'}$; $and\ a2 = a2'\frac{b}{b'}$

From the above we have a'$= a1' + a2' = a1\frac{c'}{c} + a2\frac{b'}{b}$;

And $a = a1 + a2 = a1'\frac{c}{c'} + a2'\frac{b}{b'}$;

Now, let take $\frac{a'}{a} = \frac{a1\frac{c'}{c} + a2\frac{b'}{b}}{\frac{a1'c}{c'} + \frac{a2'b}{b'}} = \frac{a1\frac{h'}{h} + a2\frac{h'}{h}}{a1'\frac{h}{h'} + a2'\frac{h}{h'}} = \frac{\frac{h'}{h}(a1+a2)}{\frac{h}{h'}(a1'+a2')} = \frac{h'^2}{h^2}\frac{a}{a'}$

Simplify the above we have, $\frac{a'}{a} = \frac{h'^2}{h^2}\frac{a}{a'}$; $or\ \frac{a'^2}{a^2} = \frac{h'^2}{h^2}$;

$Then\ can\ have\ \frac{a'}{a} = \frac{h'}{h}$;

$Finally, we\ can\ conlude$: $\frac{h'}{h} = \frac{a'}{a} = \frac{b'}{b} = \frac{c'}{c}$ = K_r

For triangle, when a line is parallel with a side, the inner triangle is considered zoomification transforming and the surface area of new triangle is also considered surface transforming with its original one with all 3 sides parallel with the outer triangle. The surface area of triangle formula is, $A = \frac{ah}{2}$ (where 'a' is the base side with the high 'h' of base 'a').

From above proof, for any new triangle forms within the original triangle can have the surface area as following,

$$A' = \frac{a'h'}{2} = \frac{(ka)(kh)}{2} = k^2\left(\frac{ah}{2}\right) = k^2(A)$$

Where k = K_r = $\frac{h'}{h} = \frac{a'}{a}$; which we will see more proofs as below.

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Parallel Transforming Percentage (PTP) Theorem

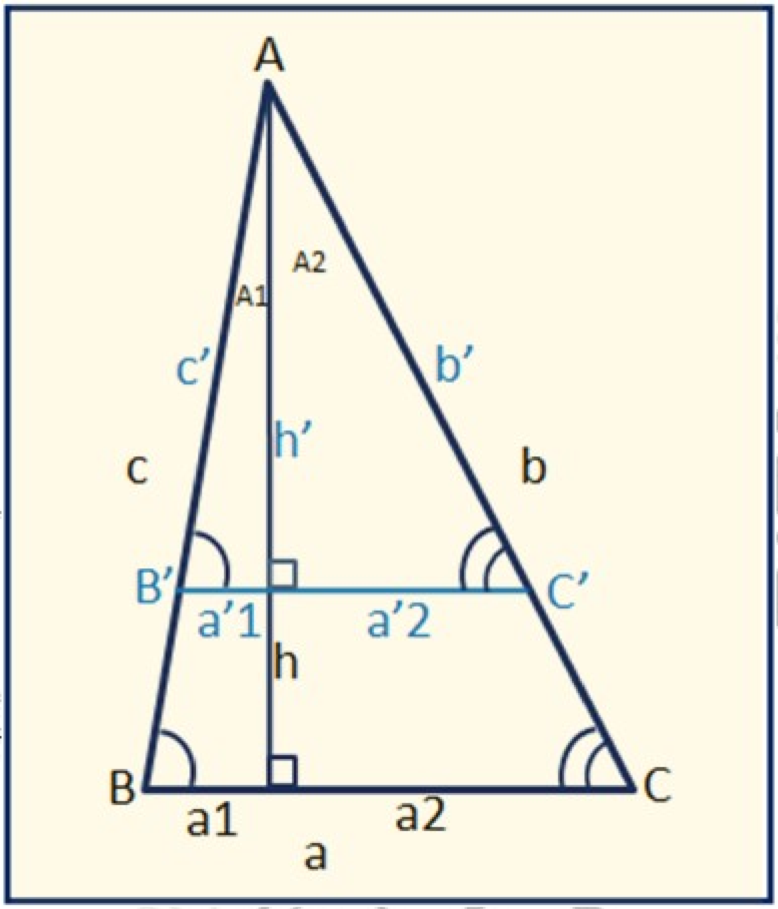

Figure-2.2: Triangle and a Parallel Transforming base line

This triangle transforming is also known as zoomification transforming within its original shape with all sides parallel and relative to the original sides, and this is true for all triangles.

Let $\mathrm{Kr} = \frac{b'}{b} = \frac{h'}{h} = \frac{c'}{c} = \frac{a'}{a}$; ($K_r$ is the key ratio)

Then $P\%(b) = b\mathrm{Kr} = b'; P\%(h) = h\mathrm{Kr} = h'; P\%(c) = c\mathrm{Kr} = c'; ect \ldots$

Circumference:

$$C = a + b + c; then$$

$$C(\mathrm{Kr}) = \mathrm{Kr}(a) + \mathrm{Kr}(b) + \mathrm{Kr}(c) = \mathrm{Kr}(a + b + c) = \mathrm{Kr} * C = P\%(C)$$

Area:

$$A = \frac{1}{2}ah; then, A(\mathrm{Kr}) = \frac{1}{2}\mathrm{Kr}(a)\mathrm{Kr}(h) = \frac{\mathrm{Kr}\text{^}2}{2}(ah) = \frac{1}{2}\mathrm{Kr}^2(A)$$

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Parallel Transforming Percentage (PTP) Theorem

Let's say we want $25\%\ of\ a; or\ \mathrm{Kr} = \frac{25}{100} = 04;$

$then = P(25\%a) = P\%(25\%b) = P(25\%c) = P(25\%h);$

Given, a= 12; $b = 25; c = 23;$ and h_a = 22.9129;

So, $a' = P(25\%a) = 3; b' = P(25\%b) = 6.25; c' = P(25\%c) =$ $5.75; and\ h' = P(25\%h) = 5.72.82;$

P$(25\%a) = \frac{25}{100}12 = 3$

We can calculate Circumference:

$\mathrm{C} = 12 + 25 + 23 = 60; Then\ C(25\%) = 3 + 6.25 + 5.75 = 15;$

Or $C(25\%) = \mathrm{Kr}(C) = \frac{25}{100}60 = 15;$

We can calculate Area as followings:

$A = \frac{1}{2}ah = \frac{1}{2}12 * 22.9129 = 137.4774;$

$$Then, A(25\%) = \frac{1}{2}\mathrm{Kr}^2 * A = \frac{1}{2}(0.4^2)(137.4774) = 10.9982;$$

Then the P%(h) is the percentage function of the high h which is also the same percentage of other side when a line is parallel transforming.

We can do the same calculation for the other 2 sides, and this proves the theorem above which stated, "*A line that parallels with a side of a triangle will divide the other 2 sides into the same percentage on both sides as well as its parallel side*".

$$C(\mathrm{Kr}) = \mathrm{Kr} * C = P\%(C)$$

$$A(\mathrm{Kr}) = \frac{1}{2}\mathrm{Kr}^2 * A$$

Additional formulas to calculate the height of each base:

$h(a) = \sqrt{b^2 - a2^2} = \sqrt{c^2 - a1^2}$; Where a = a1 + a2; or a2 = a – a1;

From above we have, $b^2 - a2^2 = c^2 - a1^2 \rightarrow b^2 - (a - a1)^2 = c^2 - a1$

After simplify we have $a1 = \frac{a^2 - b^2 + c^2}{2a};$

From $h(a) = \sqrt{c^2 - a1^2};\ Then,\ h(a) = \sqrt{c^2 - \left(\frac{a^2 - b^2 + c^2}{2a}\right)^2}$

Parallel Transforming Percentage (PTP) Theorem

Similar to other bases, we can have formulas for all 3 based heights as followings based on Figure-2.2 with the high h_a adjacent to angle A; the high h_b adjacent to angle B; and the high h_c adjacent to angle C.

$$h_a = \sqrt{c^2 - \left(\frac{a^2 - b^2 + c^2}{2a}\right)^2}$$

$$h_b = \sqrt{a^2 - \left(\frac{b^2 - c^2 + a^2}{2b}\right)^2}$$

$$h_c = \sqrt{b^2 - \left(\frac{c^2 - a^2 + b^2}{2c}\right)^2}$$

High of base a: $h_a = \sqrt{c^2 - \left(\frac{a^2 - b^2 + c^2}{2a}\right)^2}$

High of base b: $h_b = \sqrt{a^2 - \left(\frac{b^2 - c^2 + a^2}{2b}\right)^2}$

High of base c: $h_c = \sqrt{b^2 - \left(\frac{c^2 - a^2 + b^2}{2c}\right)^2}$

2. Trapezoid Parallel Transforming Percentage

Theorem-5: *A line that parallels with the bases of a trapezoid will divide the 2 sides into the same percentage on both sides, and this line is equal to the small base plus the parallel percentage of the difference of the 2 bases.*

Parallel Transforming Percentage (PTP) Theorem

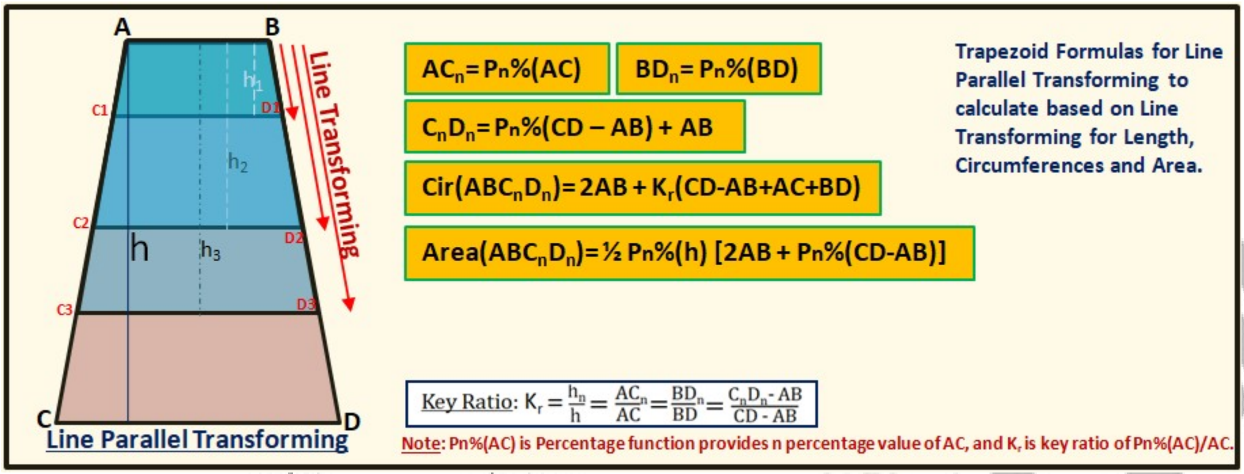

Figure-2.3: Trapezoid Base Side Parallel Transforming in Percentage

Figure-2.4 below is a detail of this figure which is used to prove this Line Parallel Transforming for trapezoid.

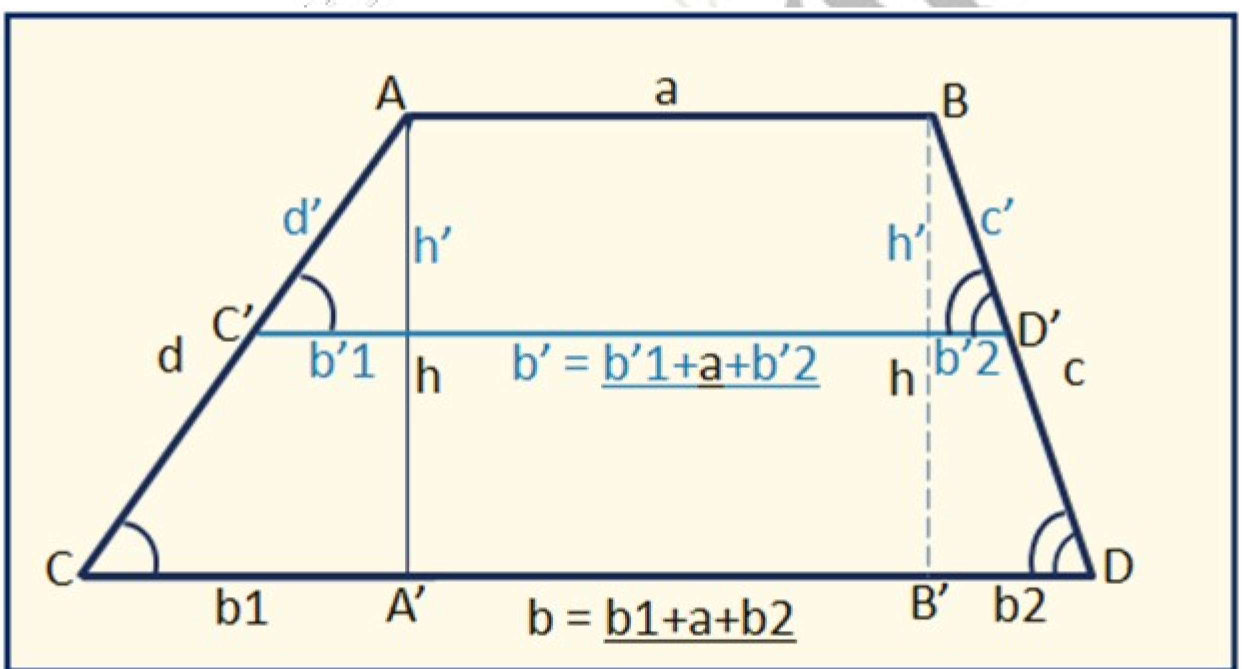

Figure-2.4: Trapezoid Base Side Parallel Transforming

Similar to the triangle proof above, from Figure-2.4 above we have,

$$\frac{d'}{d} = \frac{h'}{h} = \frac{b1'}{b1};\ And\ \frac{c'}{c} = \frac{h'}{h} = \frac{b2'}{b2}$$

Then, $b1' = b1\frac{h'}{h}$; $b2' = b2\frac{h'}{h}$

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Let take, $b1' + b2' = b1\frac{h'}{h} + b2\frac{h'}{h} = \frac{h'}{h}(b1 + b2)$

→ $\frac{h'}{h} = \frac{b1' + b2'}{(b1+b2)} = \frac{b' - a}{b-a}$

So, $\frac{h'}{h} = \frac{d'}{d} = \frac{c'}{c} = \frac{b'-a}{b-a}$ = K_r ;

Then we have,

$c' = c\,K_r$; $d' = d\,K_r$; $b' = a + (b - a)\,K_r$; $h' = h\,K_r$

We now can calculate parallel transforming Circumference of trapezoid as following,

$C' = a + b' + c' + d' = a + a + (b - a)\,K_r + c\,K_r + d\,K_r = 2a + (b - a + c + d)\,K_r$

$C' = 2a + (b - a + c + d)\,K_r$

$C' = 2a + (b - a + c + d)\,K_r$

And the parallel transforming Area,

$A' = ½\,(a + b')\,h' = ½\,(a + a + [b - a]\,K_r)\,(h\,K_r) = ½\,(2a + [b - a]\,K_r)\,(h\,K_r)$

$A' = ½\,(2a + [b - a]\,K_r)\,(h\,K_r)$

$A' = ½\,(2a + [b - a]\,K_r)\,(h\,K_r)$

Parallel Transforming Percentage (PTP) Theorem

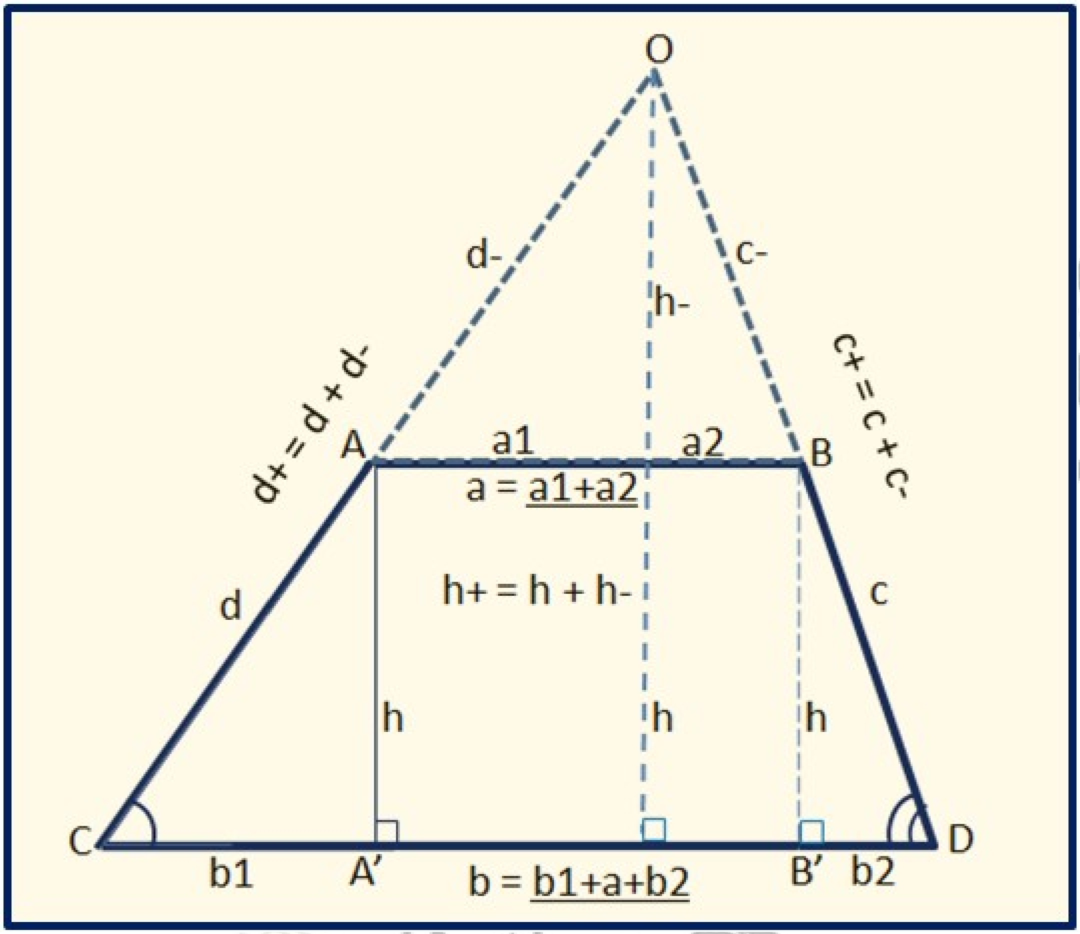

Figure-2.5: Trapezoid in Triangle Parallel Transforming

The trapezoid is the bottom part of the triangle. Apply PTP theorem from Triangle we can find the top omitted triangle OAB then we can find the trapezoid ABCD.

From Figure-2.5, we can find d-, c- and h- of the OAB triangle.

$CosD = \frac{b2}{c} = \frac{b2 + a2}{c + c-} = \frac{a2}{c-}; \quad SinD = \frac{h}{c} = \frac{h + h-}{c + c-} = \frac{h-}{c-};$

$Then, c- = c\frac{a2}{b2}; and\ h- = h\frac{c-}{c} = \frac{h}{c}\left(c\frac{a2}{b2}\right) = h\frac{a2}{b2};$

Now, let find d- of the triangle OAB.

$$CosC = \frac{b1}{d} = \frac{a1 + b1}{d + d-} = \frac{a1}{d-}; \quad SinC = \frac{h}{d} = \frac{h + h-}{d + d-} = \frac{h-}{d-};$$

$$Then, \quad d- = \frac{a1}{b1}d; \ \ h- = \frac{h}{d}d- = \frac{h}{d}\left(\frac{a1}{b1}d\right) = h\frac{a1}{b1};$$

Parallel Transforming Percentage (PTP) Theorem

$Apply\ PTP\ theorem\ for\ triangle, we\ have: \frac{a}{b} = \frac{c-}{c+c-} = \frac{d-}{d+d-};$

$Then, a(c + c -) = bc - ; \rightarrow ac\ = (b - a)c -; \rightarrow\ c- = \frac{ac}{b-a};$

$And\ we\ have, bd- = ad + ad-; \ \rightarrow\ ad = (b - a)d - ;\ d- = \frac{ad}{b-a}\ ;$

$And,\ \frac{a}{b} = \frac{h-}{h+h-}\ ;\ ah + ah- = bh-\ ; \rightarrow\ h- = \frac{ah}{b-a}$

$Finally, we\ have:$

$$h+ = h + \frac{ah}{b-a}$$

$$c+ = c + \frac{ac}{b-a}$$

$$d+ = d + \frac{ad}{b-a}$$

From those calculations, we can find the area of the big triangle A+ as follow,

$$A+ = \frac{1}{2}bh+ = \frac{1}{2}b\left(h + \frac{ah}{b-a}\right) = \frac{h}{2}\left(b + \frac{ab}{b-a}\right)$$

$$A- = \frac{1}{2}ah- = \frac{1}{2}a\left(\frac{ah}{b-a}\right) = \frac{h}{2}\left(\frac{a^2}{b-a}\right)$$

$$A+ = \frac{h}{2}\left(b + \frac{ab}{b-a}\right)$$

$$A- = \frac{h}{2}\left(\frac{a^2}{b-a}\right)$$

Then we can have the following formulas.

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Parallel Transforming Percentage (PTP) Theorem

The high of outer triangle of trapezoid: $h+ = h + \frac{ah}{b-a}$

The side c+ of outer triangle of trapezoid: $c+ = c + \frac{ac}{b-a}$

The side d+ of outer triangle of trapezoid: $d+ = d + \frac{ad}{b-a}$

The area of outer triangle of trapezoid: $A+ = \frac{h}{2}\left(b + \frac{ab}{b-a}\right)$

The area of chopped triangle of trapezoid: $A- = \frac{h}{2}\left(\frac{a^2}{b-a}\right)$

Given trapezoid with a = 15.5; b = 27; h =12; c = 14; and d = 12.743

We have,

$$h+ = h + \frac{ah}{b-a} = 12 + 15.5 * \frac{12}{27-15.5} = 28.174$$

$$c+ = c + \frac{ac}{b-a} = 14 + 15.5 * \frac{14}{27-15.5} = 32.87$$

$$d+ = d + \frac{ad}{b-a} = 12.743 + 15.5 * \frac{12.743}{27-15.5} = 29.92$$

Now we can calculate Area of outer (+) and the chopped (-) triangles as following,

$$A+ = \frac{1}{2}bh+ = \frac{1}{2}27 * 28.174 = 380.348$$

$$A- = \frac{h}{2}\left(\frac{a^2}{b-a}\right) = \frac{12}{2} * \frac{15.5^2}{27-15.5} = 125.384$$

We have the area of trapezoid is, $A = \frac{1}{2}(27 + 15.5) * 12 = 255$

We can confirm with this calculations by subtract the outer triangle area by the chopped one: $A = (A+) - (A-) = 380.384 - 125.384 = 255$

Trapezoid PTP Theorem Proof:

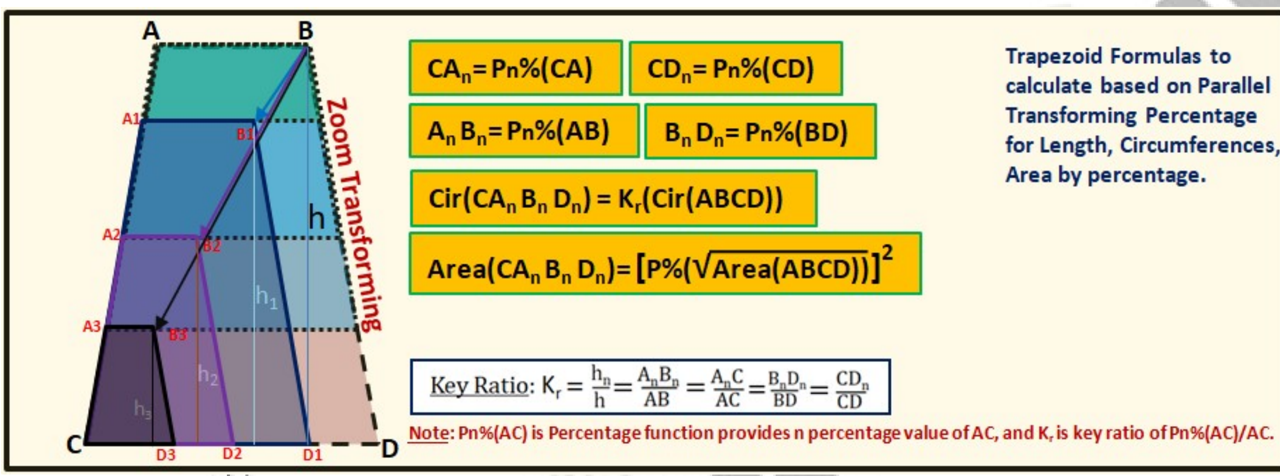

Figure-2.6: Trapezoid Parallel Transforming in Percentage

The above figure shows the trapezoid parallel transforming for all sides with the same percentage or same K_r to the C angle. For easy calculation based on the line names, let AB = a; CD = b; AC = d; BD = c; CA_n = d'; CD_n = b'; A_nB_n = a'; B_nD_n = c'. This transforming is also proved from the above proof of triangle zoomification transforming and it is the same as one side parallel with other; the trapezoid is the bottom part of the outer triangle.

Similar to the triangle, the trapezoid is the bottom part of its outer triangle. This is the zoomification transforming percentage, so all lines are relatively transforming with the same K_r. Where $K_r = \frac{a'}{a} = \frac{b'}{b} = \frac{c'}{c} = \frac{d'}{d} = \frac{h'}{h}$;

And a' = a K_r ; b' = b K_r ; c' = c K_r ; d' = d K_r

So, the trapezoid transforming Circumference can be calculated as,

$C' = a' + b' + c' + d' = a K_r + b K_r + c K_r + d K_r = (a + b + c + d) K_r = C K_r$

$C' = C K_r$; where C is the original circumference of the original trapezoid.

And the trapezoid transforming surface area can be calculated as,

$$A' = ½ (a' + b') h' = ½ (a K_r + b K_r) h K_r = ½ (a + b) h K_r = A K_r$$

$A' = A K_r$; where A is the original area of the original trapezoid.

3. Shapes & Objects Parallel Zoomification Transforming Percentage

<u>Theorem-1</u>: All spheres are parallel, and they are only different in form of percentage.

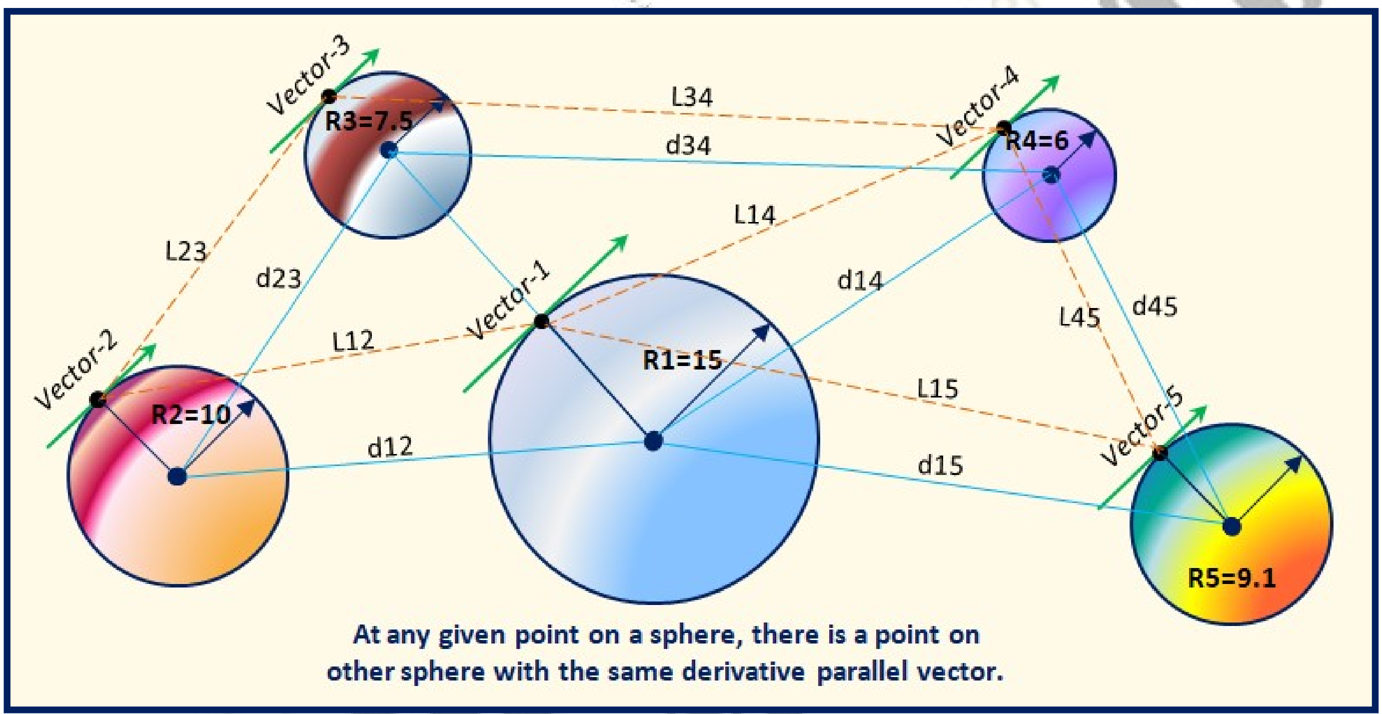

Figure-2.7: Spheres in distances

<u>Proof of theorem</u>:

At any given point on a sphere, there is always a point on other sphere with the same derivative of parallel vector as shown on the figure above. Sphere is the object that has infinite sides on any direction like circle has infinite of sides. From figure above, Vector-1 on sphere with radius R1, Vector-2 on sphere with radius R2, Vector-3 on sphere with radius R3, Vector-4 on sphere with radius R4, and Vector-5 on sphere with radius R5 are parallel with each other, and the radius line to each point of these vectors are also parallel, So all these spheres are parallel. All of these spheres are different just in percentage of the radius no matter how much difference in distance from each.

Let pick sphere with radius R1 to be the base, and we can calculate other spheres relatively to this sphere. We have sphere volume formula $V = \frac{4}{3}\pi \mathrm{R}^3$and the surface area of sphere $A = 4\pi R^2$. Let R2= k2R1; R3 = k3R1; R4=k4R1; R5 = k5R1;

Then, $A1 = 4\pi R1^2$; $V1 = \frac{4}{3}\pi \mathrm{R1}^3$

➔ $A2 = 4\pi R2^2 = 4\pi\ k2^2\ R1^2 = \mathrm{k2}^2(\mathrm{A1})$; $V2 = \frac{4}{3}\pi \mathrm{R2}^3 = \frac{4}{3}\pi\ \mathrm{k2}^2\ \mathrm{R1}^3 = \mathrm{k2}^2(\mathrm{V1})$

➔ $A3 = 4\pi R3^2 = 4\pi\ k3^2\ R1^2 = \mathrm{k3}^2(\mathrm{A1})$; $V3 = \frac{4}{3}\pi \mathrm{R3}^3 = \frac{4}{3}\pi\ \mathrm{k3}^2\ \mathrm{R1}^3 = \mathrm{k3}^2(\mathrm{V1})$

➔ $A4 = 4\pi R4^2 = 4\pi\ k4^2\ R1^2 = \mathrm{k4}^2(\mathrm{A1})$; $V4 = \frac{4}{3}\pi \mathrm{R4}^3 = \frac{4}{3}\pi\ \mathrm{k4}^2\ \mathrm{R1}^3 = \mathrm{k4}^2(\mathrm{V1})$

➔ $A5 = 4\pi R5^2 = 4\pi\ k5^2\ R1^2 = \mathrm{k5}^2(\mathrm{A1})$; $V4 = \frac{4}{3}\pi \mathrm{R5}^3 = \frac{4}{3}\pi\ \mathrm{k5}^2\ \mathrm{R1}^3 = \mathrm{k5}^2(\mathrm{V1})$

Given, R1=15; R2=10; R3=7.5; R4=6; R5=9.1;

Then $k2 = \frac{10}{15} = 0.66667 = 66.667\%$; $k3 = \frac{17.5}{15} = 0.5 = 50\%$; $k4 = \frac{6}{15} = 0.4 = 40\%$; $k5 = \frac{9.1}{15} = 0.60667 = 60.667\%$;

We have,

$A1 = 4\pi R1^2 = 4\pi\ 15^2 = 2827.4334$; $V1 = \frac{4}{3}\pi \mathrm{R1}^3 = \frac{4}{3}\pi 15^3 = 14137.1669$;
$A2 = k2^2(A1) = 0.66667^2(A1) = 1256.6496$; $V2 = k2^3(V1) = 0.66667^3(\mathrm{V1}) = 4188.8530$;
$A3 = k3^2(A1) = 0.50000^2(A1) = 706.8583$; $V3 = k3^3(V1) = 0.50000^3(\mathrm{V1}) = 1767.1489$;
$A4 = k4^2(A1) = 0.40000^2(A1) = 452.3894$; $V4 = k4^3(V1) = 0.40000^3(\mathrm{V1}) = 904.7787$;
$A5 = k5^2(A1) = 0.60667^2(A1) = 1040.6326$; $V5 = k5^3(V1) = 0.60667^3(\mathrm{V1}) = 3156.6029$;

The results are matched with the sphere formulas for each sphere.

Parallel Transforming Percentage (PTP) Theorem

Theorem-2: *All circles are parallel on the same surface or on other parallel surfaces, and they are only different in form of percentage.*

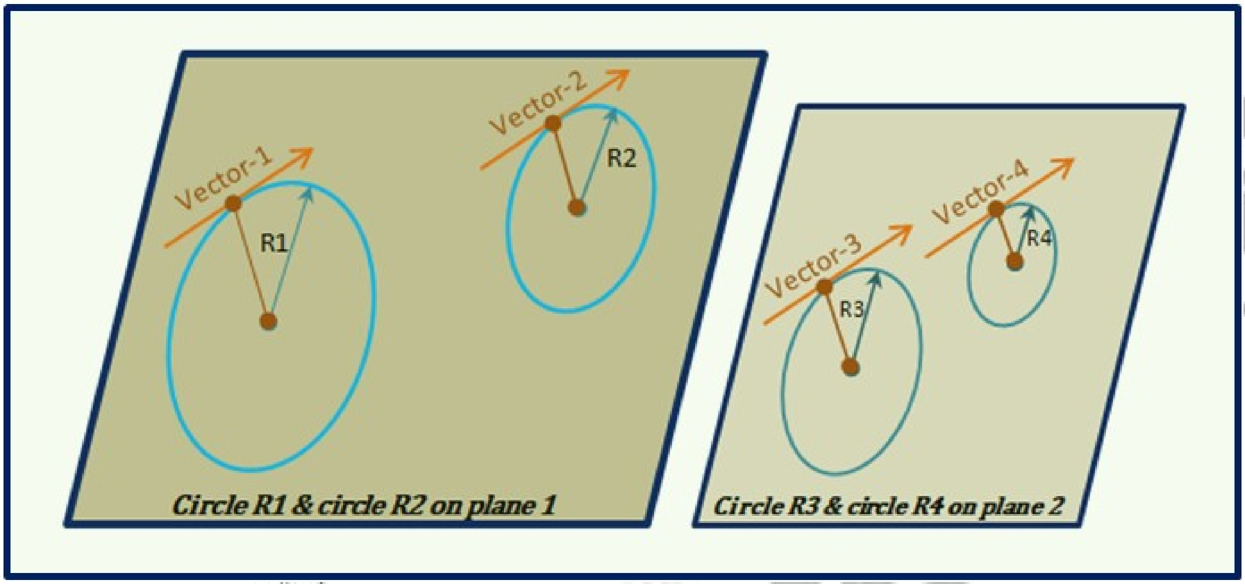

Figure-2.8: Circles in distances and on other parallel plane

Proof of theorem:

Similar to the spheres, from Figure-2.8, at any given point on a circle, there is always a point on other circle with the same derivative of parallel vector as shown on the figure below. Circle is the shape that has infinite sides. From figure below, Vector-1 on circle with radius R1 on plane-1, Vector-2 on circle with radius R2 on plane-1, Vector-3 on circle with radius R3 on plane-2, Vector-4 on circle with radius R4 on plane-2 are parallel with each other, and the radius line to each point of these vectors are also parallel, So all these circles are parallel even on other parallel planes. We have circle circumference formula $C = 2\pi R$; and area formula $A = \pi R^2$; Let R2= k2R1; R3 = k3R1; R4=k4R1;

Then we have,

$$C2 = 2\pi\, R2 = 2\pi\, k2\, R1 = k2(C1); \quad A2 = \pi\, R2^2 = k2^2(A1);$$

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$$C3 = 2\pi\ R3 = 2\pi\ k3\ R1 = k3(C1); \quad A3 = \pi\ R3^2 = k3^2(A1);$$
$$C4 = 2\pi\ R4 = 2\pi\ k4\ R1 = k4(C1); \quad A4 = \pi\ R4^2 = k4^2(A1);$$

Given, R1=15; R2=10; R3=7.5; R4=6; Then, $k2 = \frac{10}{15} = 0.66667 = 66.667\%; k3 = \frac{17.5}{15} = 0.5 = 50\%;\ k4 = \frac{6}{15} = 0.4 = 40\%;$

→ $C1 = 2\pi R1 = \ 2\pi 15 = 94.2478; A1 = \pi R1^2 = \pi\ 15^2 = 706.8583;$

→ $C2 = k2(C1) = 0.66667(C1) = 62.8322;\ A2 = \ k2^2(A1) = 0.66667^2(A1) = 314.1624;$

→ $C3 = k3(C1) = 0.50000(C1) = 47.1239;\ A3 = \ k3^2(A1) = 0.50000^2(A1) = 176.7146;$

→ $C4 = k4(C1) = 0.40000(C1) = 37.6991;\ A4 = \ k4^2(A1) = 0.40000^2(A1) = 113.0973;$

The results are matched with the formulas for each circle.

Theorem-3*:* *All lines, shapes, and objects are zoomified, are parallel transforming from its original, and they are only different in form of percentage.*

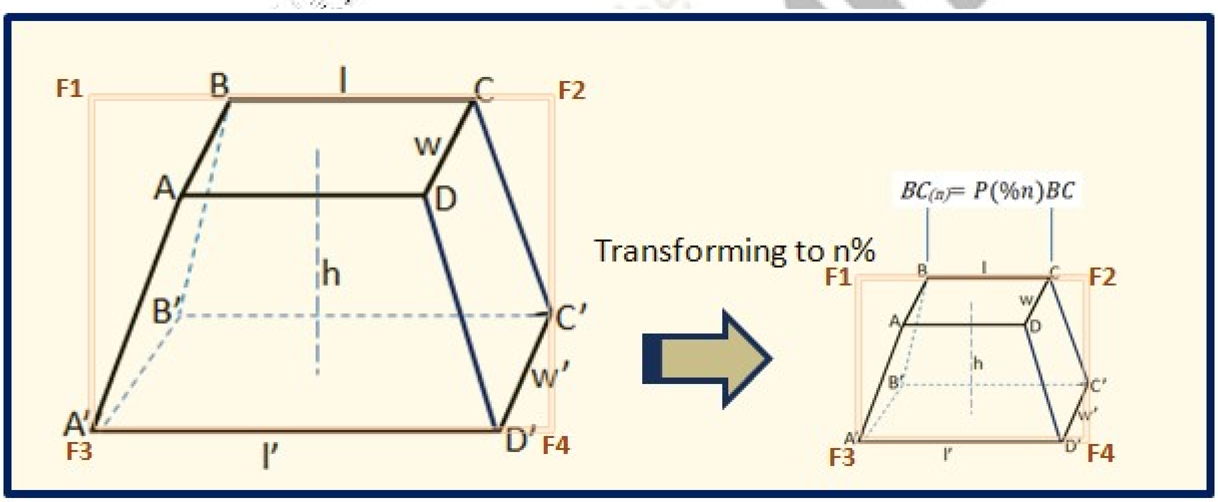

Figure-2.9: Truncated pyramid Zoomification Transforming in Percentage

<u>Proof of theorem</u>:

Figure-2.9, from the above many proofs for triangle, trapezoid, circles shapes and spheres, when the whole shape or object changing or transforming in ratio or percentage, all the lines are also changing in the same rate; and this is named as Zoomification Transforming. When zoomification transforming, the outer frame or border with 4 corners **F**1, **F**2, **F**3, **F**4 changes in n%, then all the lines are also changing by n% compare to they own original values. We have the volume and lateral area formulas of truncated pyramid as followings.

$V = \frac{\left(l'w+lw'+2\left(wl+w'l'\right)\right)h}{6}; AL = (l+l')\sqrt{\left(\frac{l'-l}{2}\right)^2 + h^2} + (w+w')\sqrt{\left(\frac{w'-w}{2}\right)^2 + h^2}$

Let the object transform to n% at key ratio K_r (short name k) we have, the new lines of new truncated pyramid $k = \frac{ln}{l} = \frac{ln'}{l'} = \frac{wn}{w} = \frac{wn'}{w'} = \frac{hn}{h}$ ... (at n is new size)

Then, $Vn = \frac{\left(kl'kw+kl\quad'+2\left(kwkl+\quad'kl'\right)\right)(kh)}{6} = k^3\frac{\left(l'w+lw'+2\left(wl+w'l'\right)\right)h}{6} = k^3(V);$

And $ALn = (kl+kl')\sqrt{\left(\frac{kl'-kl}{2}\right)^2 + k^2h^2} + (hw+hw')\sqrt{\left(\frac{hw'-hw}{2}\right)^2 + k^2h^2}$

→ $ALn = k^2\frac{\left(l'w+lw'+2\left(wl+w'l'\right)\right)h}{6};$

And at (n) new frame lines F1F2(n)=F3F4(n)=k F1F2 = k F3F4; F1F3(n)=F2F4(n)=k F1F3 = k F2F4; then the new area of the frame is $A_{Fn} = k^2F1F2 * F1F3;$

Let assume the truncated pyramid transformed to new size, and we can prove the frame must be transformed in the same percentage of K_r. We have new line F3F4(n) = kl'+D'F4(n); the lines C'D' must be parallel with its original line in the right triangle C'D'F4; and the angle D'C'F4 must be the same. So, Sin(D'C'F4) = D'F4/C'D' = D'F4/w'; the new truncate pyramid for the same angle we have Sin(D'C'F4) = x/(kw') = D'F4/w'; So, D'F4 of the new size must be (k D'F4).

Given, w=9; l=12; w'=17; l'=19; h=15; then

$$V = \frac{\left(19*9+12*17+2(9*12+19*17)\right)15}{6} = 3092.5$$

$$AL = (12+19)\sqrt{\left(\frac{19-12}{2}\right)^2 + 15^2} + (9+17)\sqrt{\left(\frac{17-9}{2}\right)^2 + 15^2} = 881.1191$$

Total Surface Area $A = AL + wl + w'l' = 881.1191 + 9*12 + 17*19 = 1312.1191$

Apply PTP theorem, we can find the new size of 40% as following by K_r = k = P(40%);

we have the new sizes, l=4.8; w=3.6; l'=7.6; w'=6.8; h=6;

$V(40\%) = k^3(V) = 0.4^3(3092.5) = 197.92;$

$A(40\%) = k^2(AL) + k^2(wl) + k^2(w'l') = 0.4^2(881.1191) + 0.4^2(9*12) + 0.4^2(17*19) = 209.9391$

These values are matched after applied the volume and area formulas based on the new lines as listed l=4.8; w=3.6; l'=7.6; w'=6.8; h=6;

4. Hollow Cylinder in Sphere Parallel Transforming Percentage

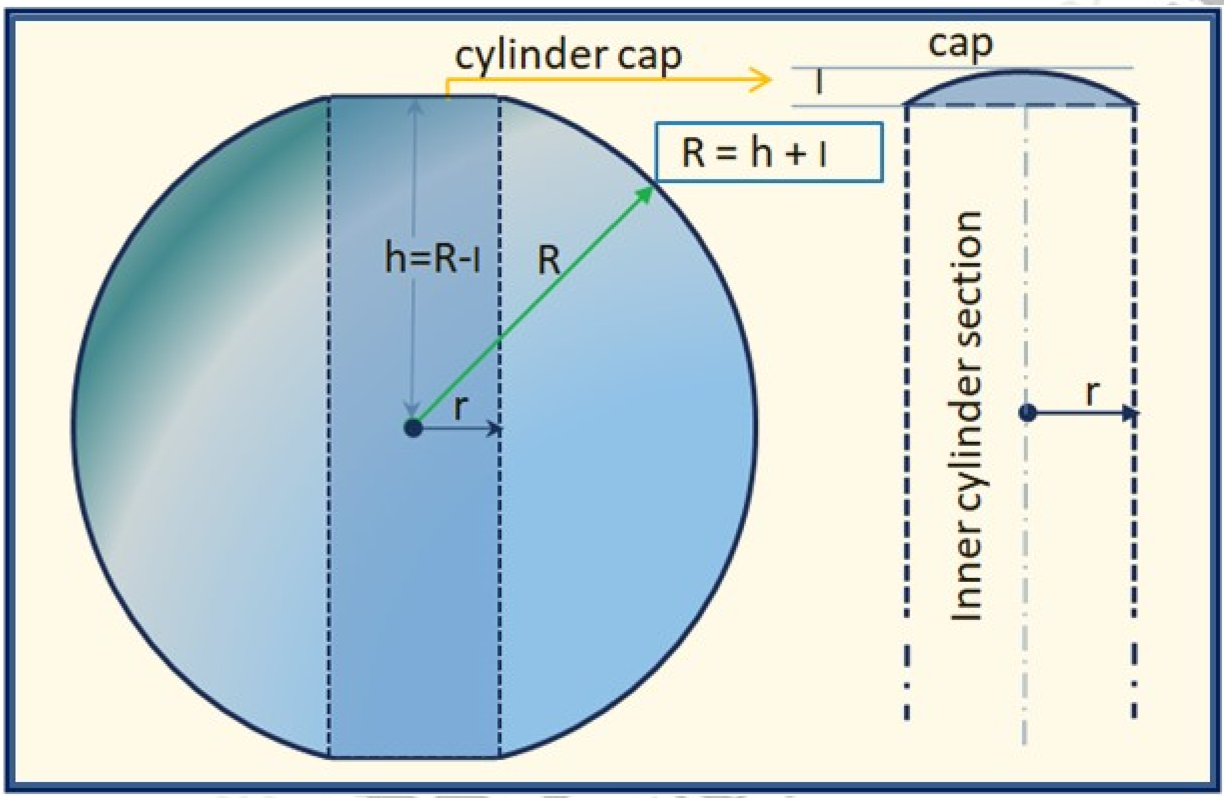

Figure-2.10: Sphere with Inner Cylinder Transforming in Percentage

From Figure-2.10, we have formulas for sphere, cylinder and sphere cap volume that we can apply to the figures below as followings.

$V_{sph} = \frac{4}{3}\pi R^3$; $V_{cyl} = \pi r^2(2h)$;

And $V_{cap} = \frac{\pi}{3} l^2(3R - l)$ or another form, $V_{cap} = \frac{1}{6}\pi l(3r^2 + l^2)$

$A_{sph} = 4\pi R^2$; $A_{Lcyl} = 2\pi r(2h)$; $A_{cap} = 2\pi rl$;

Where **R** is radius of sphere, **r** is the radius of inner cylinder with the height 2**h** = 2(**R**-**l**), and **l** is the height of the sphere cap with base radius of **r**.

Then can find transforming volumes applying PTP theorem,

$K_r = \frac{R'}{R} = \frac{r'}{r} = \frac{l'}{l} = \frac{h'}{h}$; Let $k = K_r$

$V'_{sph} = \frac{4}{3}\pi(R')^3 = \frac{4}{3}\pi(kR)^3 = k^3(\frac{4}{3} R^3)$

→ Then, $V'_{sph} = k^3 V_{sph} = (k\sqrt[3]{V_{sph}})^3$

$V'_{cyl} = \pi r'^2(2h') = \pi(kr)^2(k2h) = k^3(\pi r^2 2h)$

→ Then, $V'_{cyl} = k^3 V_{cyl} = (k\sqrt[3]{V_{cyl}})^3$

$V'_{cap} = \frac{1}{6}\pi l'(3r'^2 + l'^2) = \frac{1}{6}\pi(kl)[3(kr)^2 + (kl)^2] = k^3\left(\frac{1}{6}\pi l(3r^2 + l^2)\right)$

→ Then, $V'_{cap} = k^3 V_{cap} = (k\sqrt[3]{V_{cap}})^3$

$A'_{sph} = 4\pi(R')^2 = 4\pi(kR)^2 = k^2(4\pi R^2)$

→Then, $A'_{sph} = k^2 A_{sph} = (k\sqrt{A_{sph}})^2$

$A'_{Lcyl} = 2\pi r'(2h') = 2\pi(kr)(2kh) = k^2(2\pi r(2h))$

→Then, $A'_{Lcyl} = k^2 A_{Lcyl} = (k\sqrt{A_{Lcyl}})^2$

$A'_{cap} = 2\pi r' l' = 2\pi(kr)(kl) = k^2(2\pi rl)$

→Then, $A'_{cap} = k^2 A_{cap} = (k\sqrt{A_{cap}})^2$

Example:

Given R=75; r=18.5;

➔ $h = \sqrt{R^2 - r^2} = \sqrt{75^2 - 18.5^2} = 72.6825$

➔ $l = R - h = 75 - 72.6825 = 2.3175$

Then we have,

$\mathbf{A_{sph}} = 4\pi R^2 = 4\pi 75^2 = 70{,}685.8347$

$\mathbf{A_{Lcyl}} = 2\pi r(2h) = 2\pi * 18.5 * 2 * 72.6825 = 16{,}897.0718$

A$_{cap}$ $= 2\pi rl = 2\pi * 18.5 * 2.3175 = 269.3837$

➔ Delta Sphere Area ΔA = $A_{sph} - 2A_{cap}$

➔ **ΔA** = $70{,}685.8347 - 2(269.3837) = \mathbf{70{,}147.0673}$

V$_{sph}$ $= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi 75^3 = 1{,}767{,}145.8676$

V$_{cyl}$ $= \pi r^2(2h) = \pi 18.5^2(2 * 72.6825) = 156{,}297.9141$

V$_{cap}$ $= \frac{\pi}{3}l^2(3R - l) = \frac{\pi}{3}2.3175^2(3 * 75 - 2.3175) = 1{,}252.4321$

➔ Delta Sphere Volume ΔV = $\mathbf{V_{sph}} - \mathbf{V_{cyl}} - 2\mathbf{V_{cap}}$

➔ **ΔV** = $1{,}767{,}145.8676 - 156{,}297.9141 - 2(1{,}252.4321) = \mathbf{1{,}608{,}343.0893}$

Let R' = 30; r' = 7.4; or K_r = 30/75 = 0.4; Transform to 40%.

→ h' = 0.4(h) = 0.4(72.6825) = 29.073

→ l' = R' – h' = 30 – 29.073 = 0.927; Or l' = 0.4(l) = 0.4(2.3175) = 0.927

Then we have,

A'$_{sph}$ $= 4\pi R'^2 = 4\pi\ 30^2 = 11{,}309.7336$

A'$_{Lcyl}$ $= 2\pi r'(2h') = 2\pi * 7.4 * 2 * 29.073 = 2703.5315$

A'$_{cap}$ $= 2\pi r'l' = 2\pi * 7.4 * 0.927 = 43.1014$

ΔA' = $11{,}309.7336 - 2(43.1014) = \mathbf{11{,}223.5308}$

V'$_{sph}$ $= \frac{4}{3}\pi R'^3 = \frac{4}{3}\pi\ 30^3 = 113{,}097.3355$

V'$_{cyl}$ $= \pi r'^2(2h') = \pi 7.4^2(2 * 29.073) = 10{,}003.0665$

V'$_{cap}$ $= \frac{\pi}{3}l'^2(3R' - l') = \frac{\pi}{3}\ 0.927^2(3 * 30 - 0.927) = 80.1557$

➔ **ΔV'** = $113{,}097.3355 - 10{,}003.0665 - 2(80.1557) = \mathbf{102{,}933.9576}$

Now, can we confirm with the parallel transforming percentage for lines and the volumes from the key radio K_r = 30/75 = 0.4; Transform to 40%.

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$A'_{sph} \div A_{sph} = \frac{11{,}309.7336}{70{,}685.8347} = 0.16 = 0.4^2$

$A'_{Lcyl} \div A_{Lcyl} = \frac{2{,}703.5315}{16{,}897.0718} = 0.16 = 0.4^2$

$A'_{cap} \div A_{cap} = \frac{43{,}1014}{269.3837} = 0.16 = 0.4^2$

$\Delta A' \div \Delta A = \frac{11.223.5308}{70{,}147.0673} = 0.16 = 0.4^2$

$V'_{sph} \div V_{sph} = \frac{113{,}097.3355}{1{,}767{,}145.8676} = 0.064 = 0.4^3$

$V'_{cyl} \div V_{cyl} = \frac{10{,}003.0665}{156{,}297.9141} = 0.064 = 0.4^3$

$V'_{cap} \div V_{cap} = \frac{80.1557}{1{,}252.4321} = 0.064 = 0.4^3$

$\Delta V' \div \Delta V = \frac{102{,}933.9576}{1{,}608{,}343.0893} = 0.064 = 0.4^3$

From the above proofs, for any key ratio or percentage of transforming, we can write the new area **A'** and the new volume **V'** based on the original area **A** and volume **V** of an object as followings. Where k = K_r is the key ratio of transforming percentage.

$A' = k^2 A = (k\sqrt{A})^2$

$V' = k^3 V = (k\sqrt[3]{V})^3$

$A' = k^2 A = (k\sqrt{A})^2$ | $V' = k^3 V = (k\sqrt[3]{V})^3$

This even works for hollow sphere or hollow objects.

5. Hollow Spheres in Sphere Parallel Transforming Percentage

We have formulas for sphere, and rectangle block volume that we can apply to the figure below as followings.

$V_{sph} = \frac{4}{3}\pi R^3$; $V_{blk} = lwh$;

Where **R** is radius of sphere, and l, w, h are the length, the width and the height of the block.

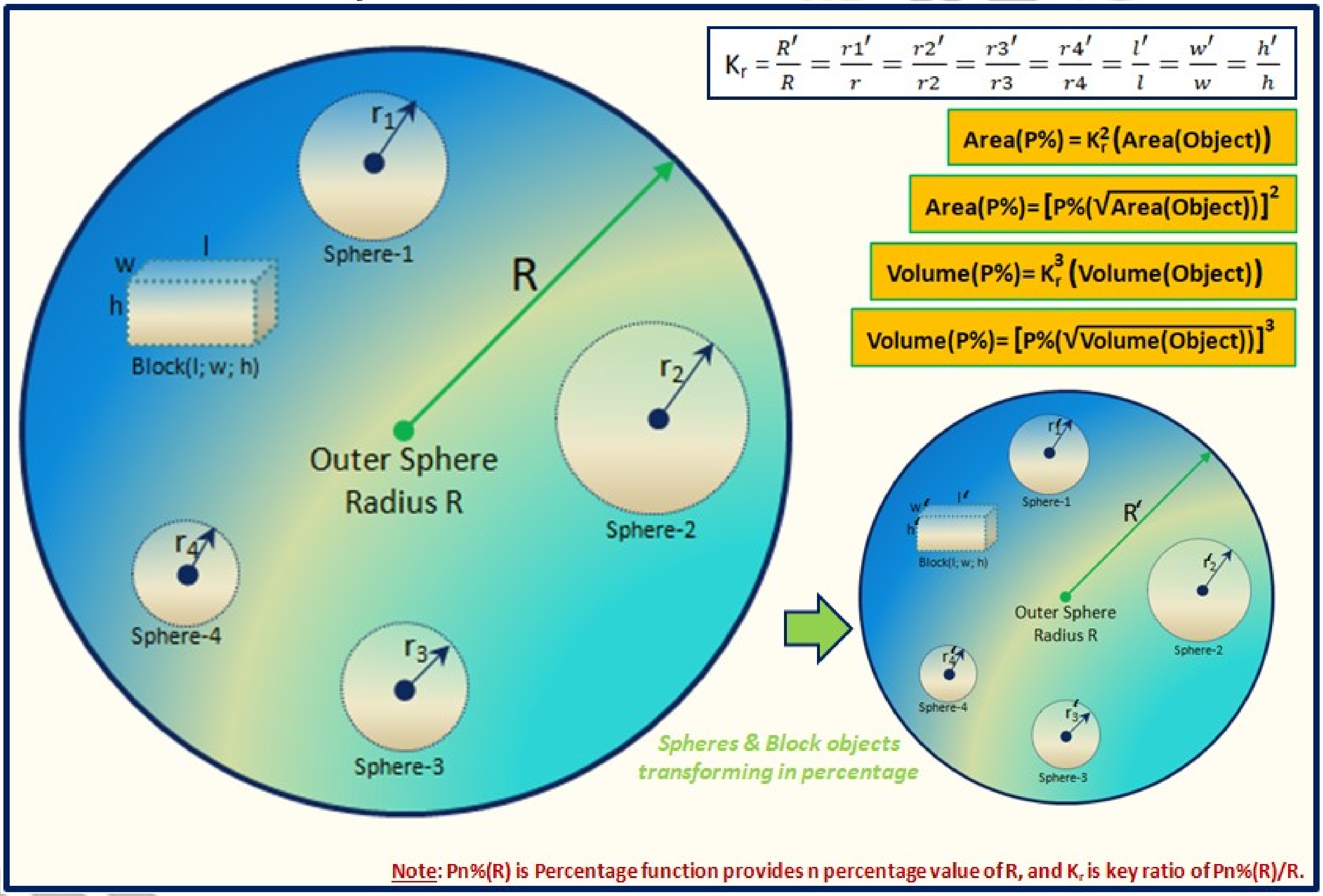

Figure-2.11: Sphere with Inner Spheres Transforming in Percentage

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Parallel Transforming Percentage (PTP) Theorem

We have $K_r = \frac{R'}{R} = \frac{r1'}{r1} = \frac{r2'}{r2} = \frac{r3'}{r3} = \frac{r4'}{r4} = \frac{l'}{l} = \frac{w'}{w} = \frac{h'}{h}$; Let $k = K_r$

The same proof as previous section, we have.

V'_{sph} $= \frac{4}{3}\pi R'^3 = \frac{4}{3}\pi k^3 R^3 = k^3 V_{sph}$**;**

V'_{blk} $= l'w'h' = klkwkh = k^3 lwh = k^3 V_{blk}$;

A'_{sph} $= 4\pi R'^2 = 4\pi k^2 R^2 = k^2 A_{sph}$ **;**

A'_{blk} $= 2(w^\wedge{}' \, l^\wedge{}' + w^\wedge{}' \, h^\wedge{}' + l^\wedge{}' \, h^\wedge{}') = 2(kwkl + kwkh + klkh) = 2k^\wedge 2 \, (wl + wh + lh) = k^2 A_{blk}$**;**

Given, R = 75; r1 = 15; r2 = 20; r3 = 10; r4 = 5; l = 12; w = 7; h = 5;

→ **V_{sph}** $= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\, 75^3 = 1{,}767{,}145.8676$

→ **V_1** $= \frac{4}{3}\pi\, r1^3 = \frac{4}{3}\pi\, 15^3 = 14{,}137.1669$;

→ **V_2** $= \frac{4}{3}\pi\, r2^3 = \frac{4}{3}\pi\, 20^3 = 33{,}510.326$;

→ **V_3** $= \frac{4}{3}\pi\, r3^3 = \frac{4}{3}\pi\, 10^3 = 4{,}188.7902$;

→ **V_4** $= \frac{4}{3}\pi\, r4^3 = \frac{4}{3}\pi\, 5^3 = 523.5988$;

→ **V_{blk}** $= lwh = 12 * 7 * 5 = 420$;

→ Delta Sphere Volume ΔV = **V_{sph} – V_1 – V_2 – V_3 – V_4 – V_{blk}**

ΔV=$1{,}767{,}145.8676 - 14{,}137.1669 - 33{,}510.326 - 4{,}188.7902 - 523.5988 - 420 = \mathbf{1{,}714{,}365.9901}$

Given, transforming down to 40% or K_r = k = 0.4;
Then, R'=30; r1'=6; r2'=8; r3'=4; r4'=2; l'=4.8; w'=2.8; h'=2;

→ **V'$_{sph}$** = $\frac{4}{3}\pi R'^3 = \frac{4}{3}\pi\ 30^3 = 113{,}097.3355$

→ **V'$_1$** = $\frac{4}{3}\pi\ r1'^3 = \frac{4}{3}\pi\ 6^3 = 904.7787;$

→ **V'$_2$** = $\frac{4}{3}\pi\ r2'^3 = \frac{4}{3}\pi\ 8^3 = 2{,}144.6606;$

→ **V'$_3$** = $\frac{4}{3}\pi\ r3'^3 = \frac{4}{3}\pi\ 4^3 = 268.0826;$

→ **V'$_4$** = $\frac{4}{3}\pi\ r4'^3 = \frac{4}{3}\pi\ 2^3 = 33.5103;$

→ **V'$_{blk}$** = $l'w'h' = 4.8 * 2.8 * 2 = 26.88;$

ΔV'= $113{,}097.3355 - 904.7787 - 2{,}144.6606 - 268.0826 - 33.5103 - 26.88 =$ $\mathbf{109{,}719.4233}$

Now, can we confirm with the parallel transforming percentage for lines and the volumes from the key radio K_r = 30/75 = 0.4; Transform to 40%.

V'$_{sph}$ ÷ V$_{sph}$ = $\frac{113{,}097.3355}{1{,}767{,}145.8676} = 0.064 = 0.4^3$

V'$_1$ ÷ V$_1$ = $\frac{904.7787}{14{,}137.1669} = 0.064 = 0.4^3$

V'$_2$ ÷ V$_2$ = $\frac{2{,}144.6606}{33{,}510.326} = 0.064 = 0.4^3$

V'$_3$ ÷ V$_3$ = $\frac{268.0826}{4{,}188.7902} = 0.064 = 0.4^3$

V'$_4$ ÷ V$_4$ = $\frac{33.5103}{523.5988} = 0.064 = 0.4^3$

V'$_{blk}$ ÷ V$_{blk}$ = $\frac{26.88}{420} = 0.064 = 0.4^3$

ΔV' ÷ ΔV = $\frac{\mathbf{109{,}719.4233}}{\mathbf{1{,}714{,}365.9901}} = 0.064 = 0.4^3$

6. Existing Ellipse Definition

Current Definition of Ellipse: "*Ellipse is a closed curve, the locus of a point such that the sum of the distances from that point to two other fixed points (called the foci of the ellipse) is constant; equivalently, the conic section that is the intersection of a cone with a plane that does not intersect the base of the cone.*"

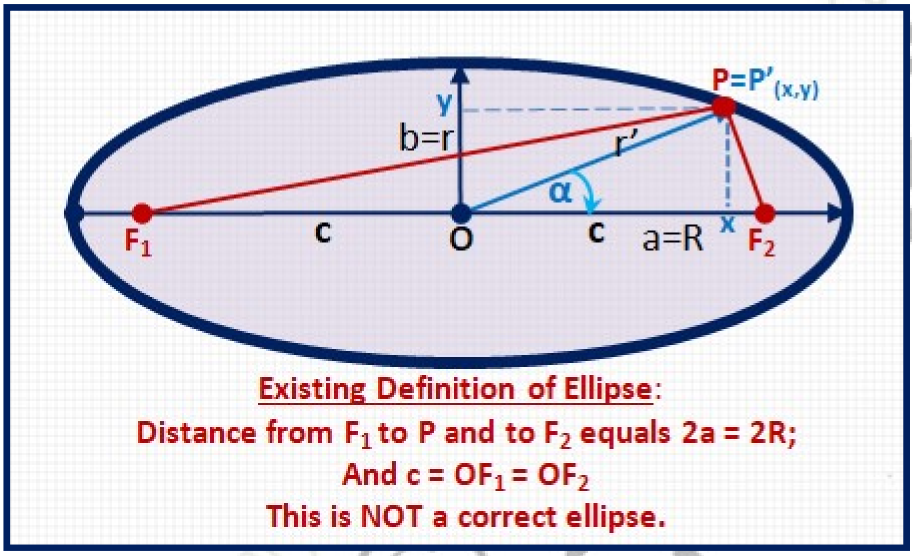

Figure-2.12: Existing Ellipse

From the existing ellipse definition, $c = \sqrt{a^2 - b^2}$;

And the definition of Eccentricity $e = \frac{c}{a} = \sqrt{1 - \left(\frac{b}{a}\right)^2}$;

The polar form radius of ellipse,

$$r(\theta) = \frac{ab}{\sqrt{\left(b\,cos(\theta)\right)^2 + (a\sin(\theta))^2}} = \frac{b}{\sqrt{1-(e\cos(\theta))^2}}$$

Using the example of ellipse above in the next section with **R** = 25; **r** = 12;

Apply Rule of Pi (π) to find the circumference of the ellipse,

$$C = 2\int_{0}^{\pi} \frac{b}{\sqrt{1-(e\cos(\theta))^2}}\, d(\theta) = 2\int_{0}^{\pi} \frac{12}{\sqrt{1-\left(1-\left(\frac{12}{25}\right)^2\right)(\cos(\theta))^2}}\, d(\theta) = 105.2734$$

This circumference value '**105.2734**' is way off from the next section of proof of new ellipse definition formulas with both set of formulas with the value of '**119.8549**'. So, the existing definition of ellipse with 2 focuses is NOT right!

7. New Ellipse Definition Based on Parallel Transforming Percentage

Theorem-6: *Ellipse is a closed curve with the perimeter that divides all the chord lines of the outer circle with radius **R** that parallel with the fixed short radius **r** of the ellipse into the same percentage on each chord line for all the chord lines with the same percentage of* $\frac{r}{R}$.

Ellipse Theorem Proof:

Figure-2.13 below shows the ellipse in its outer circle, and the followings will prove the ellipse is the outer circle rotates in certain angle on its plane. The e-chords of ellipse are relatively with the same e-chord line of the outer circle by the same percentage of the fixed short radius and the fixed long radius. The fixed long radius R is the outer circle radius.

Figure-2.14 shows the side view of the circle rotating at ϕ angle with the eye viewing on the left will see the ellipse forming on the vertical plane. The highest position at the middle on the vertical plane is the short radius **r** of the ellipse. This radius **r** is equal to **RCos(ϕ)**; and the vertical e-chords of the circle also lying on the vertical plane creating a shadow to form the e-chords of the ellipse.

Parallel Transforming Percentage (PTP) Theorem

From the Figure-2.14 below we have,

r $= R\ Cos(\phi)$; c1’ $= c1\,\mathrm{Cos}(\phi)$; c2’ $= c2\,\mathrm{Cos}(\phi)$; c3’ $= c3\,\mathrm{Cos}(\phi)$; etc…

where c1’, c2’, c3’ are the chords of the ellipse are lying on the same lines with c1, c2, c3 which are the chords of the circle; both having the same factor of Cos(ϕ). So, for any percentage of radius R or P%(R) we can have,

$$\boldsymbol{P\%(R) = \frac{c'}{c} = \frac{e'}{e} = \frac{y}{e} = \frac{r}{R}}$$

With these ratios relatively, the theorem can be proved as stated, “*The perimeter of ellipse will divide all the chord lines of the outer circle that parallel with the fixed short radius of the ellipse into the same percentage on each chord line.*”

Parallel Transforming Percentage (PTP) Theorem

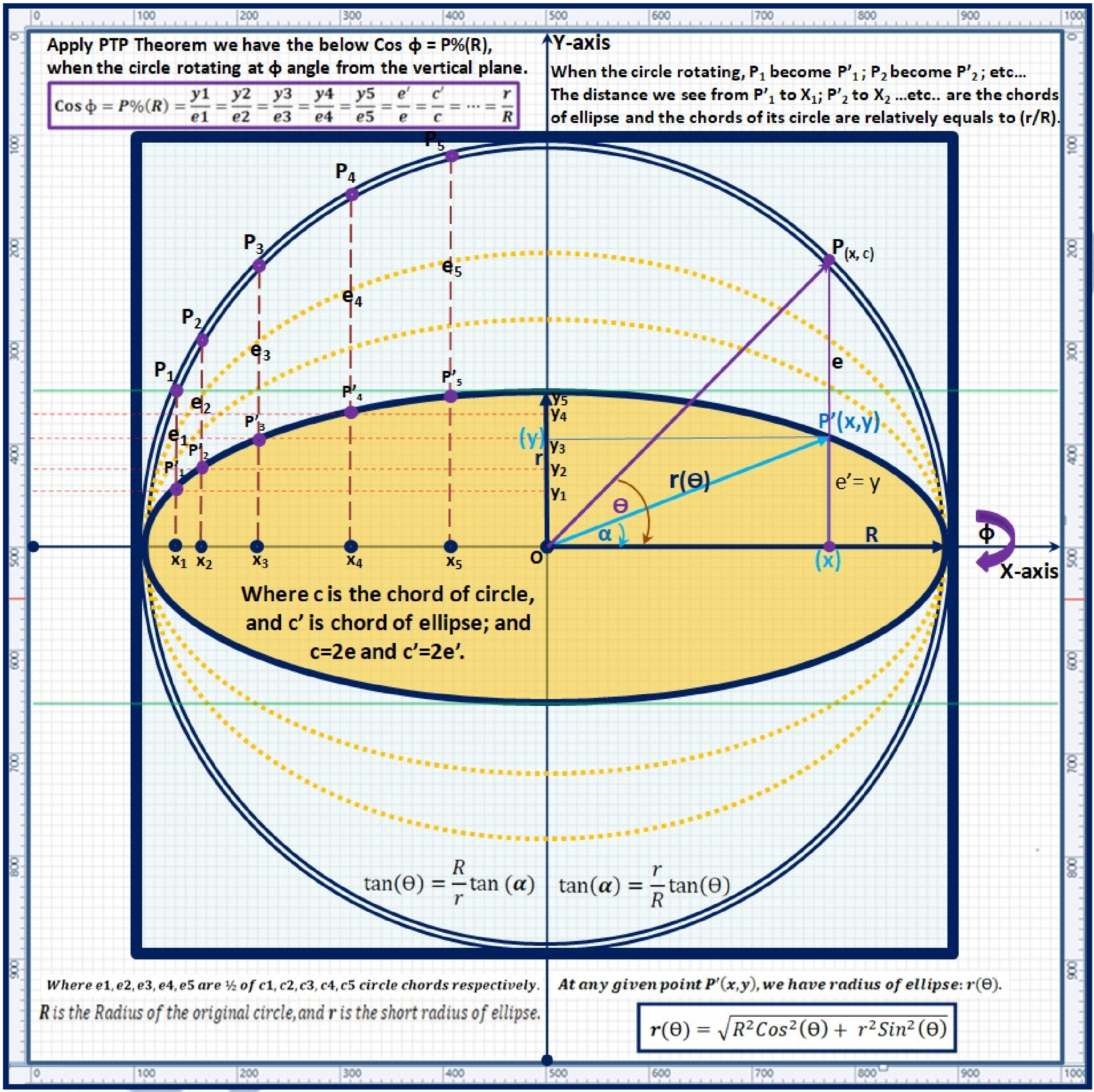

Figure-2.13: Ellipse in the outer Circle

Parallel Transforming Percentage (PTP) Theorem

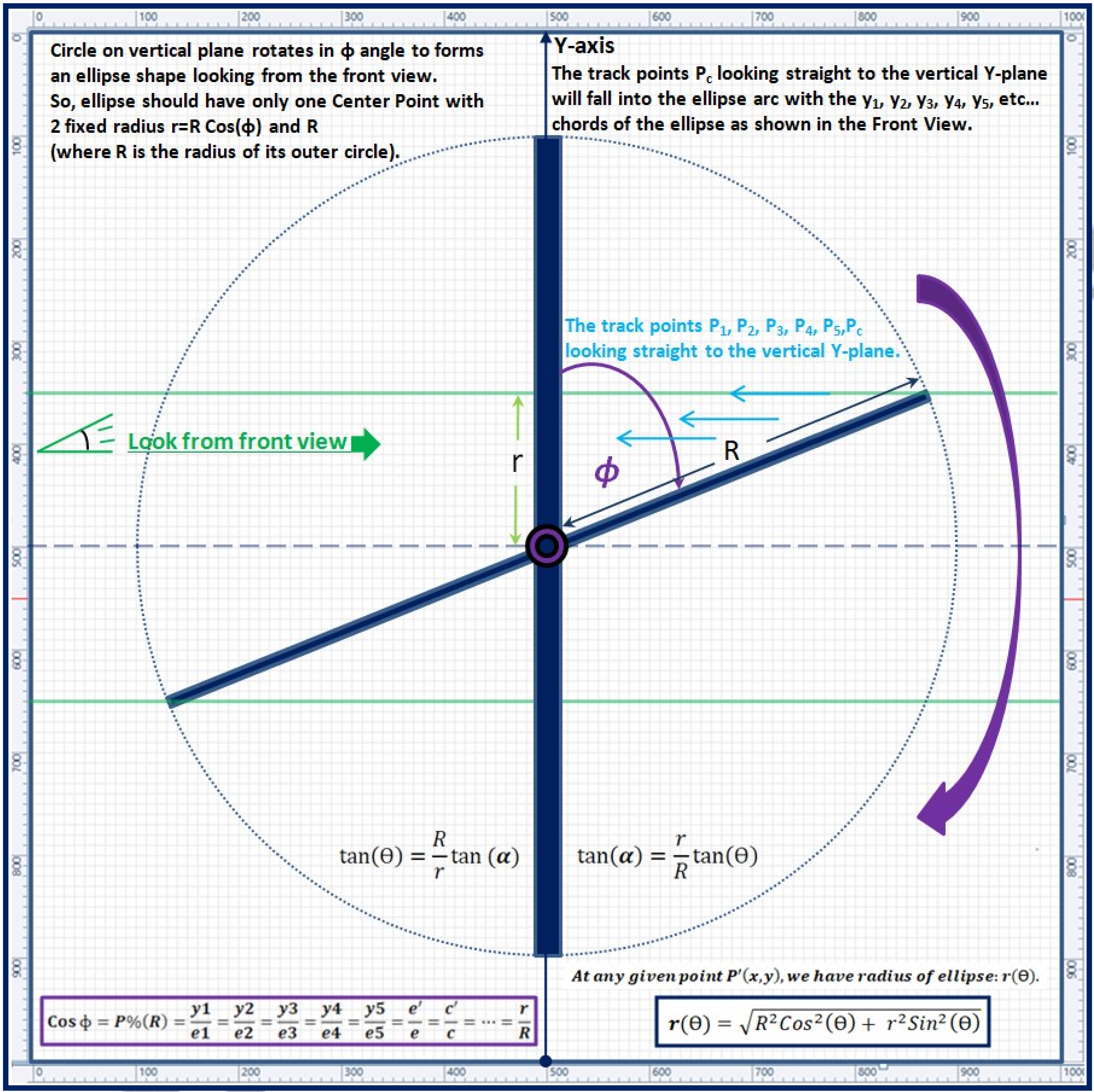

Figure-2.14: Ellipse shadow showing when the circle rotates ϕ angle

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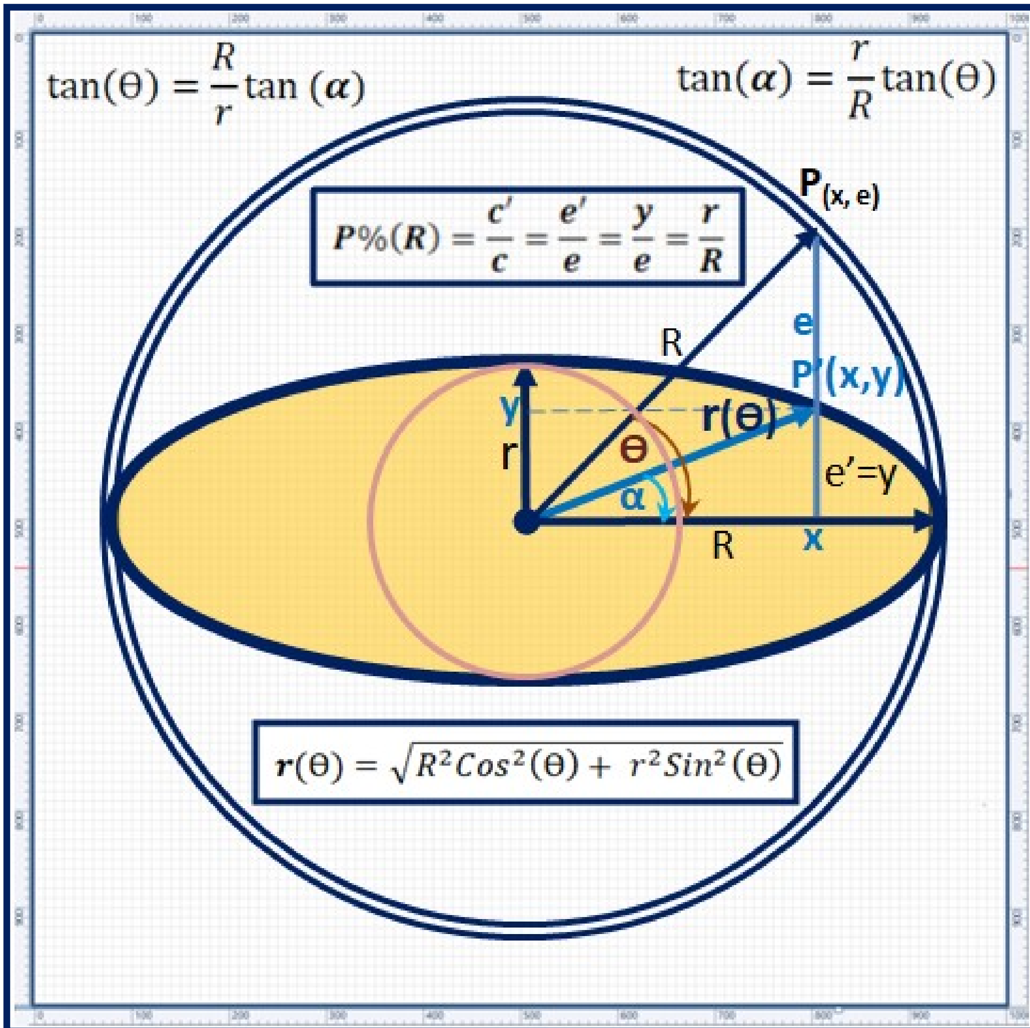

Figure-2.15: Ellipse Radius Calculations

We have, $\frac{e'}{e} = \frac{y}{e} = \frac{r}{R}$; or $\boldsymbol{y} = \boldsymbol{e}\left(\frac{r}{R}\right)$ and e $= \sqrt{R^2 - x^2}$.

Note: 'e' is ½ chord line 'c' of a circle.

$Sin(\theta) = \frac{e}{R}; where: e = \frac{R}{r}y; Then\ Sin(\theta) = \frac{1}{R}\frac{R}{r}y = \frac{y}{r};$

And, $Cos(\theta) = \frac{x}{R};$

Apply Sine and Cosine rule, we have,

$Cos(\theta)^2 + Sin(\theta)^2 = 1$; Then we have $\frac{x^2}{R^2} + \frac{y^2}{r^2} = 1$

This is the standard ellipse equation.

However, the current definition of ellipse with 2 focuses is not right!

<u>Current Definition of Ellipse</u>: "***Ellipse is a closed curve, the locus of a point such that the sum of the distances from that point to two other fixed points (called the foci of the ellipse) is constant; equivalently, the conic section that is the intersection of a cone with a plane that does not intersect the base of the cone.***"

From the above we can find the Ellipse radius $r(\Theta)$ as following:

$$r(\Theta) = \sqrt{x^2 + y^2} = \sqrt{x^2 + e^2\frac{r^2}{R^2}} = \sqrt{x^2 + \frac{r^2}{R^2}(R^2 - x^2)}$$

Where, x= $RCos(\Theta)$ then we have,

$$r(\Theta) = \sqrt{R^2Cos^2(\Theta) + \frac{r^2}{R^2}(R^2 - R^2Cos^2(\Theta))} = \sqrt{R^2Cos^2(\Theta) + r^2(1 - Cos^2(\Theta))}$$

We can finalize this equation as following with $(1 - Cos^2(\Theta)) = Sin^2(\Theta)$,

The ellipse radius: $\boldsymbol{r}(\Theta) = \sqrt{R^2Cos^2(\Theta) + r^2Sin^2(\Theta)}$

From,

$$\boldsymbol{P\%(R) = \frac{c'}{c} = \frac{e'}{e} = \frac{y}{e} = \frac{r}{R}}$$

$We\ have, y = e\frac{r}{R} = \frac{r}{R}\sqrt{R^2 - \ x^2}$

For any position of x, we can have the function of f(x) as following.

$$\boldsymbol{f(x) = \frac{r}{R}\sqrt{R^2 - x^2}}$$

$$\boxed{\boldsymbol{f(x) = \frac{r}{R}\sqrt{R^2 - x^2}}}$$

Same procedure above, we have the function of f(y) as following.

$$\boldsymbol{f(y) = \frac{R}{r}\sqrt{r^2 - y^2}}$$

$$\boxed{\boldsymbol{f(y) = \frac{R}{r}\sqrt{r^2 - y^2}}}$$

Let $u = R^2 - x^2, then\ f'^{(x)} = \frac{r}{R}\left(-\frac{x}{\sqrt{R^2-x^2}}\right) = -\frac{rx}{R\sqrt{R^2-x^2}}$.

Applying curve length integral, we have

$$C(x) = \int_{x_1}^{x_2}\sqrt{1 + f'(x)^2}dx$$

Then the circumference can be calculated as following,

$$C(x) = 4\int_{x_1}^{x_2}\sqrt{1 + \frac{r^2}{R^2}\frac{x^2}{R^2 - x^2}}dx = 4\frac{r}{R}\int_{x_1}^{x_2}\sqrt{\frac{R^2}{r^2} + \frac{x^2}{R^2 - x^2}}\ dx$$

We can have the final formula to calculate the Circumference of Ellipse as following,

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Parallel Transforming Percentage (PTP) Theorem

Circumference of Ellipse: $C(x) = \frac{4r}{R}\int_0^R \sqrt{\frac{R^2}{r^2} + \frac{x^2}{R^2-x^2}}\, d(x)$

We can calculate by **y-axis** based on the ***f(y)*** as following

Circumference of Ellipse in y-axis: $C(y) = \frac{4R}{r}\int_0^r \sqrt{\frac{r^2}{R^2} + \frac{y^2}{r^2-y^2}}\, d(y)$

Area of Ellipse formula can be calculated by applying area of curvature integral at any distance from x_1 to x_2,

$$A(x) = \int_{x_1}^{x_2} f(x)dx$$

Area of Ellipse: $A(x) = \frac{4r}{R}\int_0^R \sqrt{R^2 - x^2}\, d(x)$

Or we can calculate by **y-axis** based on the ***f(y)*** as following,

Area of Ellipse in y-axis: $A(y) = \frac{4R}{r}\int_0^r \sqrt{r^2 - y^2}\, d(y)$

Volume of Ellipse formula can be calculated by apply integral at any distance from x_1 to x_2,

$$V(x) = \int_{x_1}^{x_2} f(x)^2 \pi\, dx$$

Volume of Ellipse: $V(x) = \frac{2\pi r^2}{R^2}\int_0^R (R^2 - x^2)\, d(x)$

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Note that this Volume formula also works for ellipsoid with 2 different small radius r which means r1 and r2 and r1 < R and r2 <R. This would be the ellipsoid with long radius R with different high and thickness of r1 and r2.

Calculate Circumference, Area and Volume of Ellipse and Ellipsoid with Polar method in θ angle. We have radius of Ellipse at any given θ angle, and the relationship between θ and α angles.

The ellipse radius: $\boldsymbol{r}(\Theta) = \sqrt{R^2Cos^2(\Theta) + r^2Sin^2(\Theta)}$

From the above of $\frac{\boldsymbol{e'}}{\boldsymbol{e}} = \frac{\boldsymbol{y}}{\boldsymbol{e}} = \frac{\boldsymbol{r}}{\boldsymbol{R}}$; we can calculate the relationship of $\boldsymbol{\alpha}$ and $\boldsymbol{\theta}$ angles as following,

$\tan(\alpha) = \frac{e'}{x}$; And $\tan(\theta) = \frac{c}{x}$; so, $\frac{e'}{\tan(\alpha)} = \frac{e}{\tan(\theta)}$;

Or, $\frac{\tan(\boldsymbol{\alpha})}{\tan(\boldsymbol{\theta})} = \frac{\boldsymbol{e'}}{\boldsymbol{e}} = \frac{\boldsymbol{r}}{\boldsymbol{R}}$

Relationship between α and Θ angles: $\tan(\boldsymbol{\alpha}) = \frac{r}{R}\tan(\Theta)$; $\tan(\Theta) = \frac{R}{r}\tan(\boldsymbol{\alpha})$

Apply **Rule of $\boldsymbol{\pi}$**, we have

Circumference of Ellipse by angle: $\boldsymbol{C}(\Theta) = 2\int_0^{\pi}\sqrt{\boldsymbol{R}^2\boldsymbol{Cos}^2(\Theta) + \boldsymbol{r}^2\boldsymbol{Sin}^2(\Theta)}\,\boldsymbol{d}(\Theta)$

Area of Ellipse by angle: $$A(\Theta) = 2\pi \int_{0}^{\pi/2} \left(R\,Sin(\Theta) * r\,Cos(\Theta)\right) d(\Theta)$$

Volume of Ellipsoid by angle: $$V(\Theta) = 4\pi \int_{0}^{\pi/2} \left(R\,Sin(\Theta) * r^2 Cos^2(\Theta)\right) d(\Theta)$$

Note that this Volume formula also works for ellipsoid with 2 different small radius r which means r1 and r2 and r1 < R and r2 <R. This would be the ellipsoid with long radius R with different high and thickness of r1 and r2.

Apply Polar coordinate for, x = R Sin θ and y = y Cos θ.

$$\frac{dx}{d\theta} = (R\ Cos\theta)' = -R\ Sin\theta;\ and\ \frac{dy}{d\theta} = (r\ Sin\theta)' = r\ Cos\theta;$$

Then the ellipse Circumference can be calculated as following,

Then Arc Length,

$$Arc\ Length = \int_{\Theta_1}^{\Theta_2} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\ d(\theta)$$

$$Ellipse\ Arc\ Length\ C(\Theta) = \int_{\Theta_1}^{\Theta_2} \sqrt{R^2 Sin^2(\Theta) + r^2 Cos^2(\Theta)}\ d(\Theta)$$

$$C(\Theta) = 4\int_{0}^{\pi/2} \sqrt{R^2 Sin^2(\Theta) + r^2 Cos^2(\Theta)}\ d(\Theta)$$

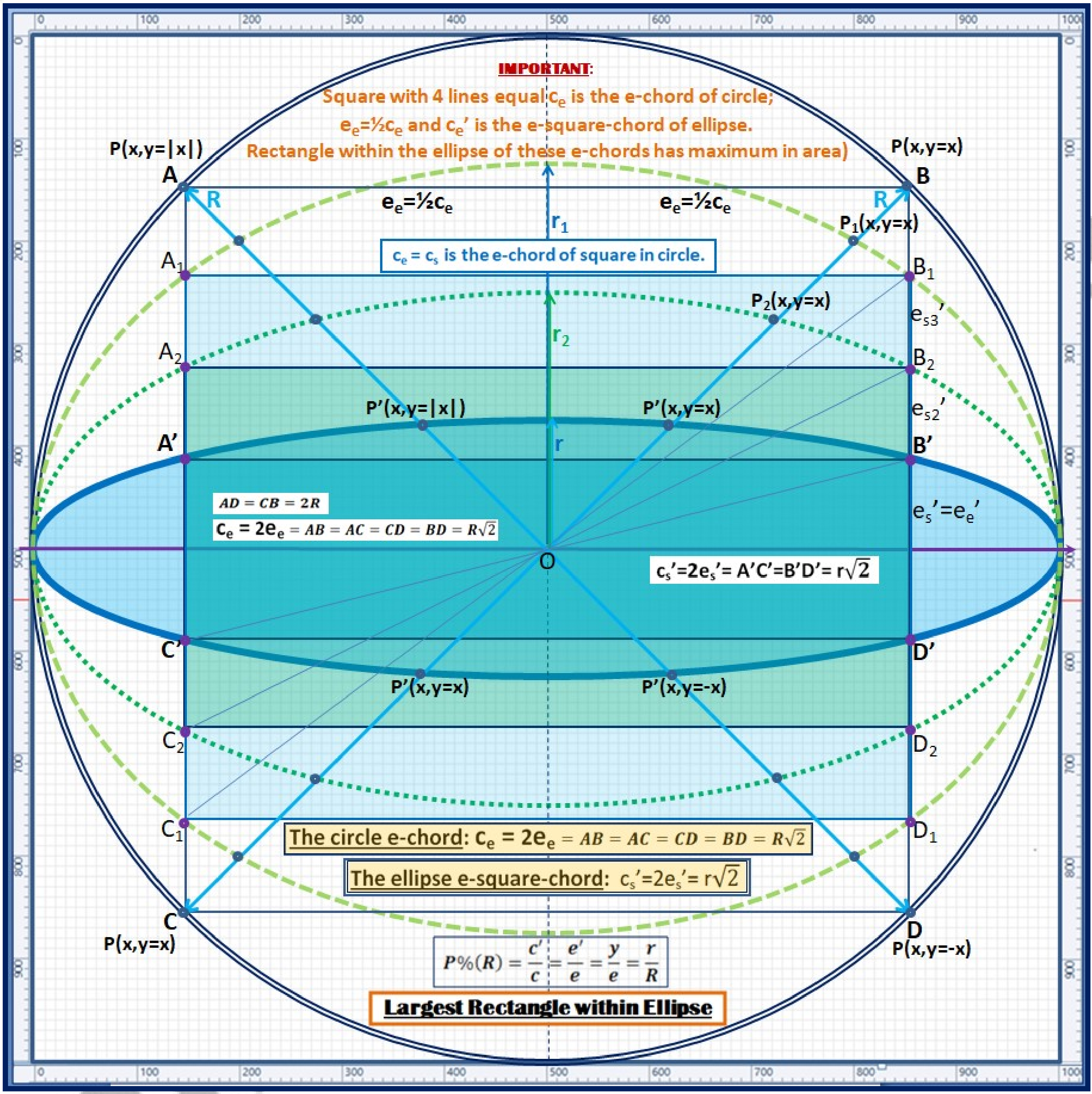

Figure-2.16: Ellipse Largest Rectangle A'B'C'D'

From the Additional Theorems & Proofs for the e-chord and e-chord largest shape in a circle, the square ABCD is the largest square in the circle with radius R. The ellipse is the circle rotates at an angle on its plane, and the rectangle within the e-chord square of the outer circle must be the largest rectangle of the ellipse. Please see the proof of Largest Rectangle in ellipse in the Additional Theorems & Proofs section. As proved, the rectangle with width is $\sqrt{2}R$, and the high is $\sqrt{2}r$.

The e-chord rectangle diagonal: $d_{r\text{-}max} = \sqrt{2(R^2 + r^2)}$

The e-chord rectangle Area: $A_{r\text{-}max} = L \times W = (R\sqrt{2}) \times (r\sqrt{2}) = 2Rr$

To calculate the average circumference of the ellipse without using integral, we can take the average approximation of the largest rectangle diagonal line as the largest diameter, and the smallest diameter of small radius r plus the largest radius R as shown below.

$$C \approxeq \frac{\pi(R+r) + \pi\sqrt{c^2 + c'^2}}{2} = \frac{\pi(R+r) + \pi\sqrt{2R^2 + 2r^2}}{2}$$

$$\mathbb{C} \approxeq \frac{\pi(R+r) + \pi\sqrt{2(R^2+r^2)}}{2}; \left(\mathbb{C} \text{ is more accurate when } P\%(R) = \frac{r}{R} \geq 0.2 \text{ or } 20\%\right)$$

$$\mathbb{C} \approxeq \frac{\pi(R+r) + \pi\sqrt{2(R^2+r^2)}}{2}$$

<u>Note</u>: ≊ is the average approximation symbol.

<u>Example</u>:

Find ellipse Circumference, Area and Ellipsoid for ellipse with **R** = 25; **r** = 12; using both set of formulas.

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Parallel Transforming Percentage (PTP) Theorem

$$C(x) = \frac{4r}{R}\int_0^R \sqrt{\frac{R^2}{r^2} + \frac{x^2}{R^2 - x^2}}\, d(x) = \frac{4(12)}{25}\int_0^{25} \sqrt{\frac{25^2}{12^2} + \frac{x^2}{25 - x^2}}\, d(x) = 119.8549$$

$$C(y) = \frac{4R}{r}\int_0^r \sqrt{\frac{r^2}{R^2} + \frac{y^2}{r^2 - y^2}}\, d(y) = \frac{4(25)}{12}\int_0^{12} \sqrt{\frac{12^2}{25^2} + \frac{y^2}{12^2 - y^2}}\, d(y) = 119.8549$$

$$C(\theta) = 2\int_0^{\pi} \sqrt{R^2 Cos^2(\theta) + r^2 Sin^2(\theta)}\, d(\theta) = 2\int_0^{\pi} \sqrt{25^2 Cos(\theta)^2 + 12^2 Sin(\theta)^2}\, d(\theta) = 119.8549$$

For each arc length, we can use this formula:

$$\text{Ellipse Arc Length } C(\theta) = \int_{\theta_1}^{\theta_2} \sqrt{R^2 Sin^2(\theta) + r^2 Cos^2(\theta)}\, d(\theta)$$

$$Arc1\ C(\theta) = \int_0^{\pi/4} \sqrt{25^2 Sin(\theta)^2 + 12^2 Cos(\theta)^2}\, d(\theta) = 11.798$$

$$Arc2\ C(\theta) = \int_{\pi/4}^{\pi/3} \sqrt{25^2 Sin(\theta)^2 + 12^2 Cos(\theta)^2}\, d(\theta) = 5.525$$

$$Arc3\ C(\theta) = \int_{\pi/3}^{\pi/2} \sqrt{25^2 Sin(\theta)^2 + 12^2 Cos(\theta)^2}\, d(\theta) = 12.641$$

Adding up these Arc lines together will be the Circumference:

$$Complete\ Arc\ Circumference = 4\,(11.798 + 5.525 + 12.641) = 119.856$$

Notice: There is the relationship between $\boldsymbol{\alpha}$ and $\boldsymbol{\theta}$ depends on which we want to calculate the arc at which angle.

Now we can calculate the average approximation of ellipse circumference without using integral from the above formula.

Parallel Transforming Percentage (PTP) Theorem

$$\mathbb{C} \approxeq \frac{\pi(R+r)+\pi\sqrt{2(R^2+r^2)}}{2} = \frac{\pi(25+12)+\ \pi\sqrt{2\left(25^2+12^2\right)}}{2} = 119.7219$$

This average approximation is every close to the one taking integral above. We can use this approach for school for younger student that are not ready for integral yet.

$$Ellipse\ Arc\ Length\ C(\Theta) = \int_{\theta_1}^{\theta_2}\sqrt{R^2Sin^2(\Theta)+\ r^2Cos^2(\Theta)}\ d(\Theta)$$

$$A(x) = \frac{4r}{R}\int_0^R\sqrt{R^2-x^2}\ d(x) = \frac{4(12)}{25}\int_0^{25}\sqrt{25^2-\ x^2}\ d(x) = 942.4778$$

$$A(y) = \frac{4R}{r}\int_0^r\sqrt{r^2-y^2}\ d(y) = \frac{4(25)}{12}\int_0^{12}\sqrt{12^2-\ y^2}\ d(y) = 942.4778$$

$$A(\Theta) = 2\pi\int_0^{\pi/2}\left(R\,Sin(\Theta)*r\,Cos(\Theta)\right)d(\Theta) = 2\pi\int_0^{\pi/2}\left(25\,Sin(\theta)*12\,Cos(\theta)\right)d(\theta) = 942.4778$$

This value of area is also correct for the linear formula:

A = $\boldsymbol{\pi rR = \ \pi 12 * 25 = 942.4778}$

$$V(x) = \frac{2\pi r^2}{R^2}\int_0^R(R^2-x^2)\ d(x) = 2\pi\frac{12^2}{25^2}\int_0^{25}(25^2-x^2)\ d(x) = 15079.6447$$

$$V(\Theta) = 4\pi\int_0^{\pi/2}\left(R\,Sin(\Theta)*\ r^2Cos^2(\Theta)\right)d(\Theta) = 4\pi\int_0^{\pi/2}(25\,Sin(\theta)*12^2Cos(\theta)^2)\ d(\theta) = 15079.6447$$

Parallel Transforming Percentage (PTP) Theorem

III. Additional Theorems & Proofs

1. E-Chord of Circle

The e-chord Theorem:

Theorem-7: *The e-chords are the* ***n*** *lines adjacent to the* ***n*** *equal angles in a circle where* ***n>2***, *are equal to* ***2*** *times of the circle radius multiply by* ***sine*** *of* ***π/n***.

The e-chord length formula: $C_{e(n)} = 2R\ Sin\left(\frac{\pi}{n}\right)$

The e-chord theorem proof:

The c_e is the e-chord, and e_e is ½ of the e-chord of a circle which can be proved as below.

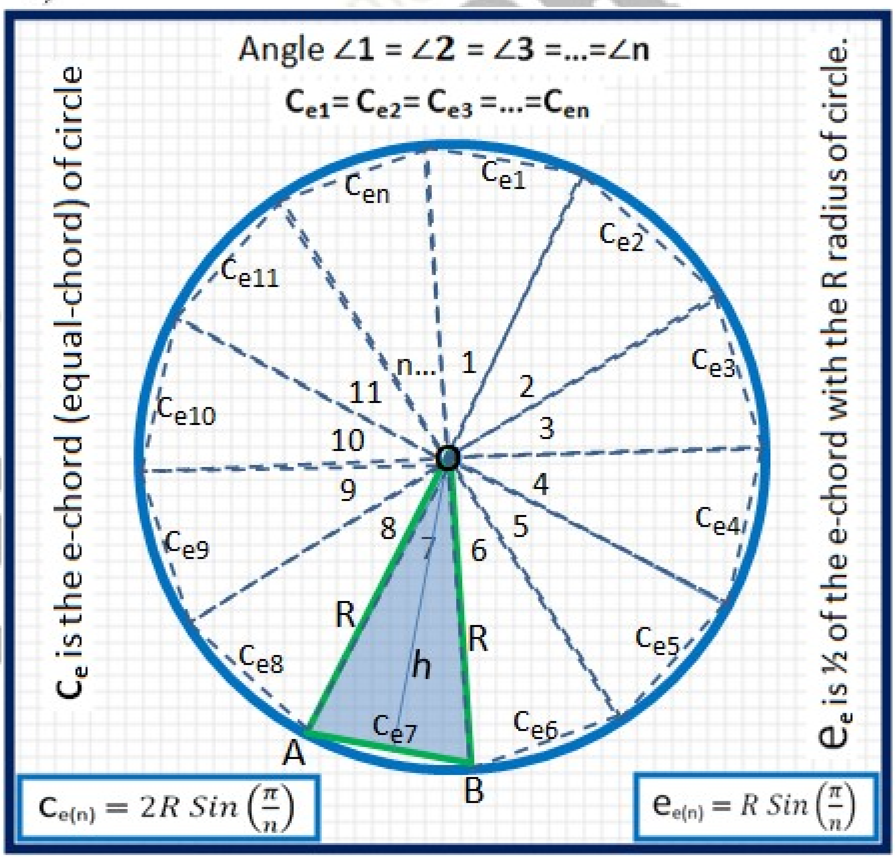

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Figure-3.1: The e-chord C_e of circle

$$C_{e(n)} = 2R\ Sin\left(\frac{\pi}{n}\right) \qquad e_{e(n)} = R\ Sin\left(\frac{\pi}{n}\right)$$

To prove these formulas, we can easily divide the circle into **n** equal angles with each angle equals to $(2\pi/n)$. Then each triangle has 2 sides equal radius R, and we can divide this triangle by half to have 2 right triangles to calculate the e-chord AB as shown in the figure above by, $AB = 2R * Sin\left(\frac{2\pi}{2n}\right) = 2R * Sin(\frac{\pi}{n})$. Where the triangle AOB has the line AB is the equal-chord $\mathbf{C_e}$ of circle as shown above, and the high h = R $Cos(\frac{\pi}{n})$.

2. The largest e-chord Shape in Circle

<u>**The e-chord shape Theorem**</u>:

<u>Theorem-8</u>: *The e-chord shape with all **n** sides equal is the largest shape compares to other shapes with the same number of **n** chord lines that are not equal in a circle, and the shape with more e-chord lines has larger area.*

The e-chord sector area formula: $A_{en} = R^2 Sin\left(\frac{\pi}{n}\right) Cos\left(\frac{\pi}{n}\right) = \frac{1}{2} R^2 Sin\left(2\frac{\pi}{n}\right)$

<u>**The e-chord shape theorem proof**</u>:

The e-chord shape with all **n** sides equal will cover largest area compare to other shapes with the same number **n** sides in a circle which can be proved as below.

For every e-chord sector triangle we have,

E-Chord length: $C_{e(n)} = 2R\ Sin\left(\frac{\pi}{n}\right)$

E-Chord triangle high: $h_{e(n)} = R\,Cos\left(\frac{\pi}{n}\right)$

Let n=12 and the circle radius of R=100 and each angle is 30°, then we have each e-chord sector shape with equal area and equal to Area = 100^2 Sin($\frac{\pi}{12}$) Cos($\frac{\pi}{12}$) = 2,500 by applying the following formula for each sector shape area,

$A_{en} = R\,Sin\left(\frac{\pi}{n}\right)\sqrt{R^2 - R^2 Sin^2\left(\frac{\pi}{n}\right)} = R^2 Sin\left(\frac{\pi}{n}\right) Cos\left(\frac{\pi}{n}\right)$

Now let sector-7 bigger than sector-8 by 1° and keep the rest the same, then we can calculate area for both shapes as below for both sectors.

$A_{e7} = 100^2 Sin\left(\frac{31°}{2}\right) Cos\left(\frac{31°}{2}\right)$ = 2,575.190375

$A_{e8} = 100^2 Sin\left(\frac{29°}{2}\right) Cos\left(\frac{29°}{2}\right)$ = 2,424.048101

However, we the sum area of these 2 sector shapes is (2,575.190375 + 2,424.048101 = 4,999.238476) which is less than sum of 2 e-chord sector of 5,000.

Now let sector-7 bigger than sector-8 by 15° and keep the rest the same, then we can calculate area for both shapes as below for both sectors.

$A_{e7} = 100^2 Sin\left(\frac{45°}{2}\right) Cos\left(\frac{45°}{2}\right)$ = 3,535.533906

$A_{e8} = 100^2 Sin\left(\frac{15°}{2}\right) Cos\left(\frac{15°}{2}\right)$ = 1,294.095226

However, we the sum area of these 2 sector shapes is (3,535.533906 + 1,294.095226 = 4,829.629131) which is less than sum of 2 e-chord sector of 5,000 and even less than 4,999.238476 the 1° difference above. So, these 2 sectors area leave more spaces uncovered compare to the e-chord sector areas.

Now let sector-7 bigger than sector-8 by 25° and keep the rest the same, then we can calculate area for both shapes as below for both sectors.

$A_{e7} = 100^2 Sin\left(\frac{55°}{2}\right) Cos\left(\frac{55°}{2}\right) = 4,095.760221$

$A_{e8} = 100^2 Sin\left(\frac{5°}{2}\right) Cos\left(\frac{5°}{2}\right) = 435.778714$

However, we the sum area of these 2 sector shapes is (4,095.760221 + 435.778714 = 4,531.538935) which is less than sum of 2 e-chord sector of 5,000 and even less than 4,829.629131 the 15° difference above. So, these 2 sectors area leave even more spaces uncovered compare to the e-chord sector areas. This proves that the e-chord shape will always greater than other shapes with the same number of sides, and the more number of sides the more coverage of circle area with the e-chord shape. This is the reason why the circle is the best ever shape that has minimum in circumference but greatest in area. Note that circle is a special shape with infinity of sides. Apply mathematic surface rotation methodology to this proof, we also conclude additional volume to surface ratio of the objects with 'n' sides equal have larger volume to surface ratio compare to other objects with 'n' sides not equal; and of course sphere is the magic object has infinite lines equals has the greatest volume to surface ratio compare to other objects. This is an additional conclusion to the above proofs but not on the theorems, so it is not necessary to have a detail of volume to surface ratio proof.

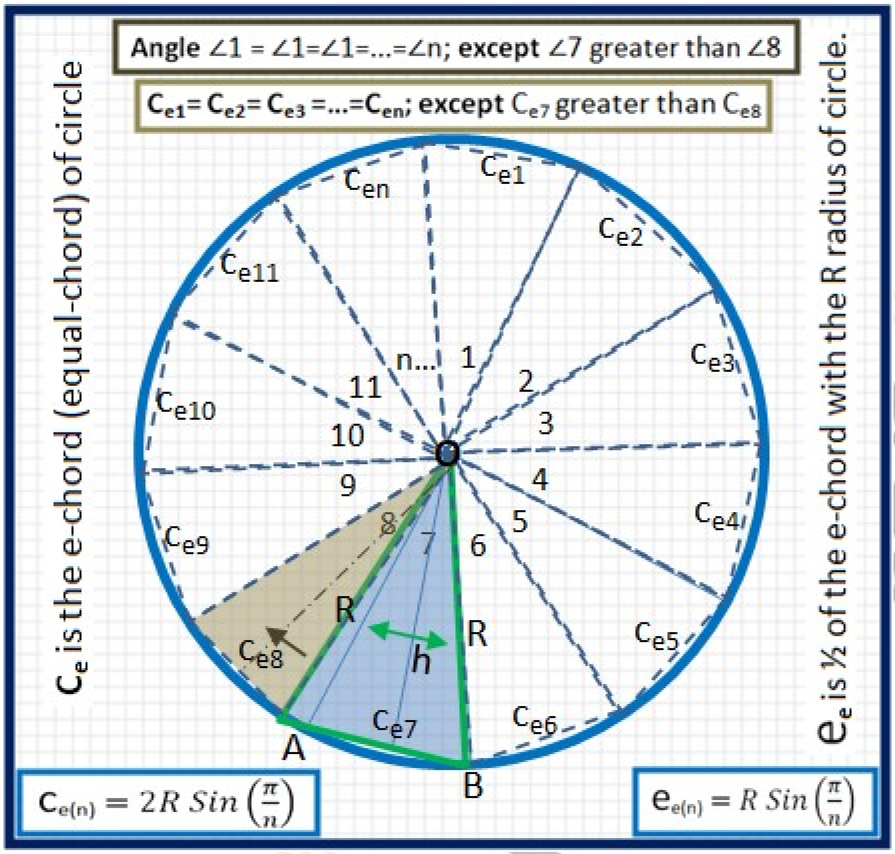

Figure-3.2: The largest e-chord shape of circle

From the above proof, when we have the largest triangle must be the equilateral triangle within a circle will have 3 lines equal to $\mathbf{C_{e3}} = 2\text{R Sin}\left(\frac{\pi}{3}\right)$; and the largest square within a circle will have 4 lines equal to $\mathbf{C_{e4}} = \sqrt{2}R$ which is $\mathbf{C_{e4}} = 2\text{R Sin}\left(\frac{\pi}{4}\right)$; etc...

We can also calculate the ratio of the area verse the circumference of the shape to find the shape that has greatest area with smallest in circumference.

We have circumference and area of an e-chord shape as following,

$\mathbf{C}_{e(n)} = 2R\,Sin\left(\frac{\pi}{n}\right)$; and $\mathbf{A}_{e(n)} = R^2 Sin\left(\frac{\pi}{n}\right) Cos\left(\frac{\pi}{n}\right)$

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Let $\mathbf{A}_{e(n)} \div \mathbf{C}_{e(n)} = \frac{R^2 Sin\left(\frac{\pi}{n}\right)Cos\left(\frac{\pi}{n}\right)}{2R\ Sin\left(\frac{\pi}{n}\right)} = \frac{RCos\left(\frac{\pi}{n}\right)}{2}$

We can compare, $\mathbf{A}_{e(n)} \div \mathbf{C}_{e(n)}$ with $\mathbf{A}_{e(n-1)} \div \mathbf{C}_{e(n-1)}$ as following.

$RCos\left(\frac{\pi}{n}\right)$; And $RCos\left(\frac{\pi}{n-1}\right)$; For n>2, we always have $Cos\left(\frac{\pi}{n}\right) > Cos\left(\frac{\pi}{n-1}\right)$;

So, $RCos\left(\frac{\pi}{n}\right) > RCos\left(\frac{\pi}{n-1}\right)$

Then, $\mathbf{A}_{e(n)} \div \mathbf{C}_{e(n)}$ is always greater than $\mathbf{A}_{e(n-1)} \div \mathbf{C}_{e(n-1)}$ for n > 2. So, the more e-chord lines of a shape, the more area ratio we have for that shape. This is also true for all shapes which have more equal lines will have greater in area ratio. The circle has infinite lines, so circle has greatest area ratio.

We can also prove when a chord is not equal to other e-chords, then at least we have 1 chord smaller and 1 chord larger than the e-chord. Now we can compare total area of these two not equal triangles to the sum of the other 2 equal areas as following.

Let A_e is the e-chord area, and A- is the area of a smaller chord, and A+ is the area of the larger chord. Then we can prove $2A_e > (A- + A+)$;

We have $A_e = R^2 Sin\left(\frac{\pi}{n}\right) Cos\left(\frac{\pi}{n}\right) = \frac{1}{2}R^2\ Sin(2\frac{\pi}{n})$; or $2A_e = R^2\ Sin(2\frac{\pi}{n})$;

Let x is the different angle between these 2 triangles,

Then $A- = \frac{1}{2}R^2\ Sin(2\frac{\pi}{n} - 2x)$; And $A+ = \frac{1}{2}R^2\ Sin(2\frac{\pi}{n} + 2x)$;

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We now have, $A- + A+ = \frac{1}{2}R^2\text{Sin}\left(2\frac{\pi}{n} - 2x\right) + \frac{1}{2}R^2\text{Sin}\left(2\frac{\pi}{n} + 2x\right) = \frac{1}{2}R^2\left(\text{Sin}\left(2\frac{\pi}{n}\right)\text{Cos}(2x) + \text{Cos}\left(2\frac{\pi}{n}\right)\text{Sin}(2x) + \text{Sin}\left(2\frac{\pi}{n}\right)\text{Cos}(2x) - \right.$ Cos2πnSin2x=R2(Sin2πnCos2x) ;

Now, let compare $2A_e > (A- + A+)$ we have,

$R^2\ \text{Sin}(2\frac{\pi}{n}) > R^2(\text{Sin}\left(2\frac{\pi}{n}\right)\text{Cos}(2x))$

→ 1 **>** Cos(2x); This is true as we have $2\frac{\pi}{n}$ **>** 2x **>** $\frac{\pi}{n}$ for n > 2 and Cos(2x) must be less than 1.

3. The largest Rectangle of Ellipse

<u>The largest rectangle in ellipse theorem</u>:

<u>Theorem-9</u>: *The rectangle in an ellipse which belongs to the e-chord square of the outer circle of the ellipse, has the largest area with the width equals to* $\sqrt{2}$ *multiply by the fixed long radius* ***R*** *and the height equals to* $\sqrt{2}$ *multiply by the fixed short radius* ***r*** *of the ellipse.*

The e-chord rectangle Area: $A_{r\text{-}max} = L \times W = (R\sqrt{2}) \times (r\sqrt{2}) = 2Rr$

The e-chord rectangle diagonal: $d_{r\text{-}max} = \sqrt{2(R^2 + r^2)}$

<u>The largest rectangle in ellipse theorem proof</u>:

The e-chord-rectangle of the ellipse has greatest area compare to other rectangles within the ellipse which can be proved as below.

Parallel Transforming Percentage (PTP) Theorem

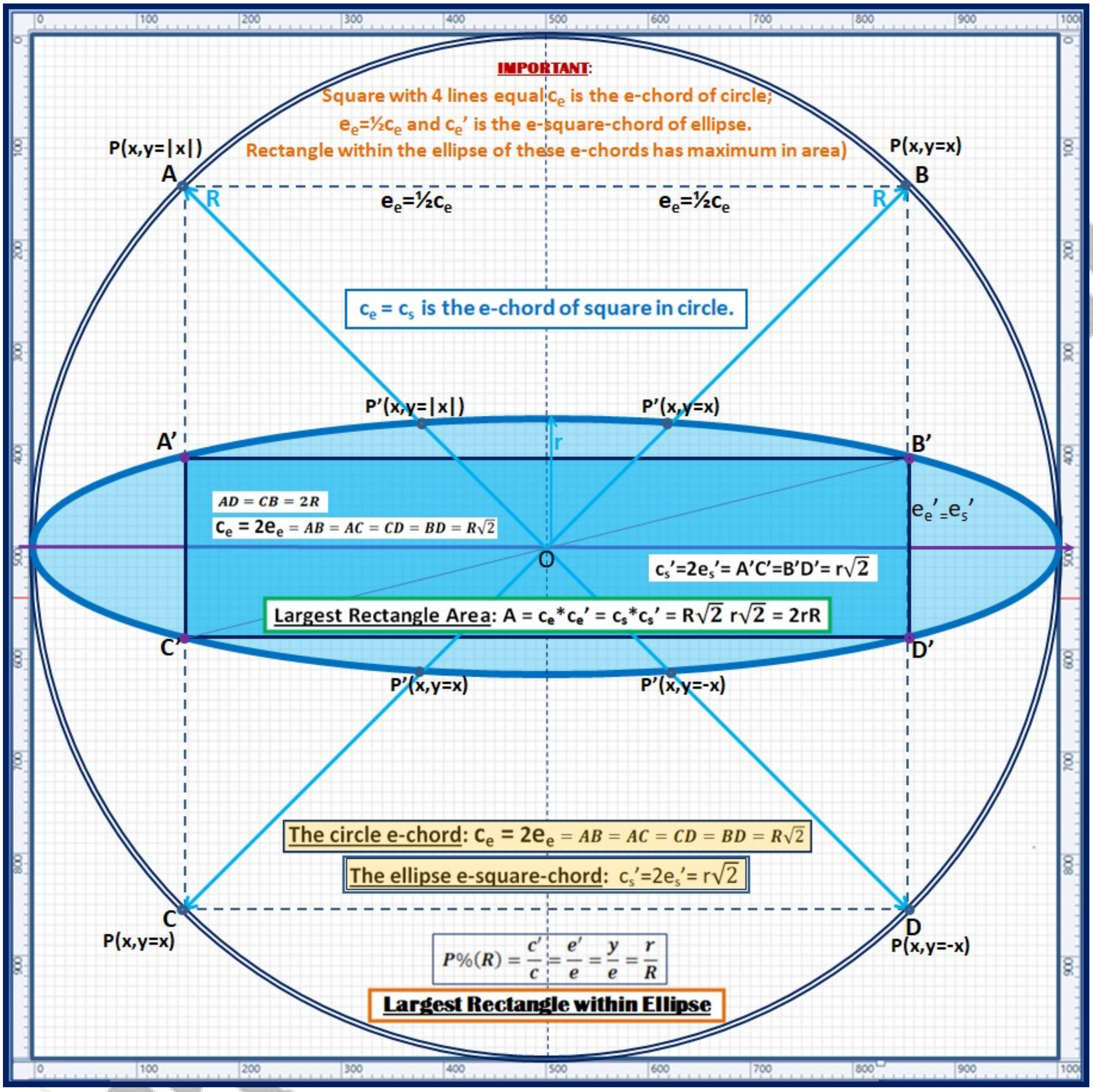

Figure-3.3: The largest Rectangle in Ellipse

To prove rectangle A'B'C'D' is the largest rectangle which is the shape within the largest square of the outer circle with radius R, in the ellipse. Let P%(R) = 25%; with R=100, and r=25;

Then the largest Area of rectangle $\mathbf{A_{max}}$ = A'B' * B'D' = c_e*c'_e = $R\sqrt{2}$ * $r\sqrt{2}$ = 2rR = 2(100 * 25) = **5000**;

Now, let A'B' = $R\sqrt{2}$ -1 = $100\sqrt{2}$ - 1 = 140.421356 or x=70.210678;

Then apply the above proof, $f(x) = \frac{r}{R}\sqrt{R^2 - x^2}$;

We have $f(x) = y = \left(\frac{25}{100}\right)\sqrt{100^2 - 70.210678^2} = 17.801792$ and compare to the e-chord $y_e = r\sqrt{2} = 25\sqrt{2}$ = 17.677670;

We now have new area of rectangle (L-1) at P%(R) = 25%:

A- = 2x*2y = 2(70.210678) * 2(17.801792) = **4,999.503350**; which is smaller than the maximum area rectangle with value of $\mathbf{A_{max}}$**= 5,000**.

Now let A'B' = $R\sqrt{2}$ + 1 = $100\sqrt{2}$ + 1 = 142.421356 or x=71.210678;

Then find $f(x) = y = \left(\frac{25}{100}\right)\sqrt{100^2 - 71.210678^2} = 17.551178$ and compare to e-chord $y_e = r\sqrt{2} = 25\sqrt{2}$ = 17.677670;

We now have new area of rectangle (L+1) at P%(R) = 25%**:**

$\mathbf{A^+}$ = 2x*2y = 2(71.210678) * 2(17.551178) = **4,999.496425**; which is smaller than the maximum area rectangle with value of $\mathbf{A_{max}}$**= 5,000**.

Let's try with P%(R)=50%; with R=100, and r=50; we have the maximum rectangle area equals $\mathbf{A_{max}}$**= 10,000**.

We have new area of rectangle (L-1) at P%(R) = 50%**:** Using the same calculation,

A- = 2x*2y=2(70.210678)*2(35.603584) = **9,999.006992**; which is smaller than the maximum area rectangle with value of $\mathbf{A_{max}}$**= 10,000**.

We have new area of rectangle (L+1) at P%(R) = 50**%:** Using the same calculation,

A^+ = 2x*2y=2(71.210678)*2(35.103559) = **9,998.992849**; which is smaller than the maximum area rectangle with value of **A_{max}= 10,000**.

We have new area of rectangle (L-1) at P%(R) = 75**%:** Using the same calculation,

A- = 2x*2y=2(70.210678)*2(53.405375) = **14,998.510489**; which is smaller than the maximum area rectangle with value of **A_{max}= 15,000**.

We have new area of rectangle (L+1) at P%(R) = 75**%:** Using the same calculation,

A^+ = 2x*2y=2(71.210678)*2(52.655338) = **14,998.489274**; which is smaller than the maximum area rectangle with value of **A_{max}= 15,000**.

This proof with 4 corners percentage checking based on small radius r vs. big radius R. So, all ellipses will follow into this rule with maximum rectangle area of **A = 2rR** with the length equals **R**$\sqrt{\mathbf{2}}$ and the width equals **r**$\sqrt{\mathbf{2}}$;

We can also prove in another way,

From the above prove, we have x = R Cos(θ); and y = r Sin(θ);

The rectangle area of in ellipse is,

$$A = 4xy = 4RrCos(\theta)Sin(\theta) = 2RrSin(2\theta);$$

The largest value of Sine is the $\frac{\pi}{2}$ angle. So, $\theta = \frac{\pi}{4}$; this rectangle with the width equals 2x = 2R Cos($\frac{\pi}{4}$) = 2R($\frac{1}{\sqrt{2}}$) = R$\sqrt{\mathbf{2}}$ and the height equals 2y = 2r Sin($\frac{\pi}{4}$) = r$\sqrt{\mathbf{2}}$ has largest area which falls within the e-chord square of the outer circle.

The e-chord rectangle diagonal: $d_{r\text{-}max} = \sqrt{2(R^2 + r^2)}$

The e-chord rectangle Area: $A_{r\text{-}max} = L \times W = (R\sqrt{2}) \times (r\sqrt{2}) = 2Rr$

IV. Apply the theorem in real life

1. Find Surface Center of Triangle

Apply PTP theorem to find the surface center of triangle, the surface center is also known as the mass center of the triangle of the same material with the same thickness and density.

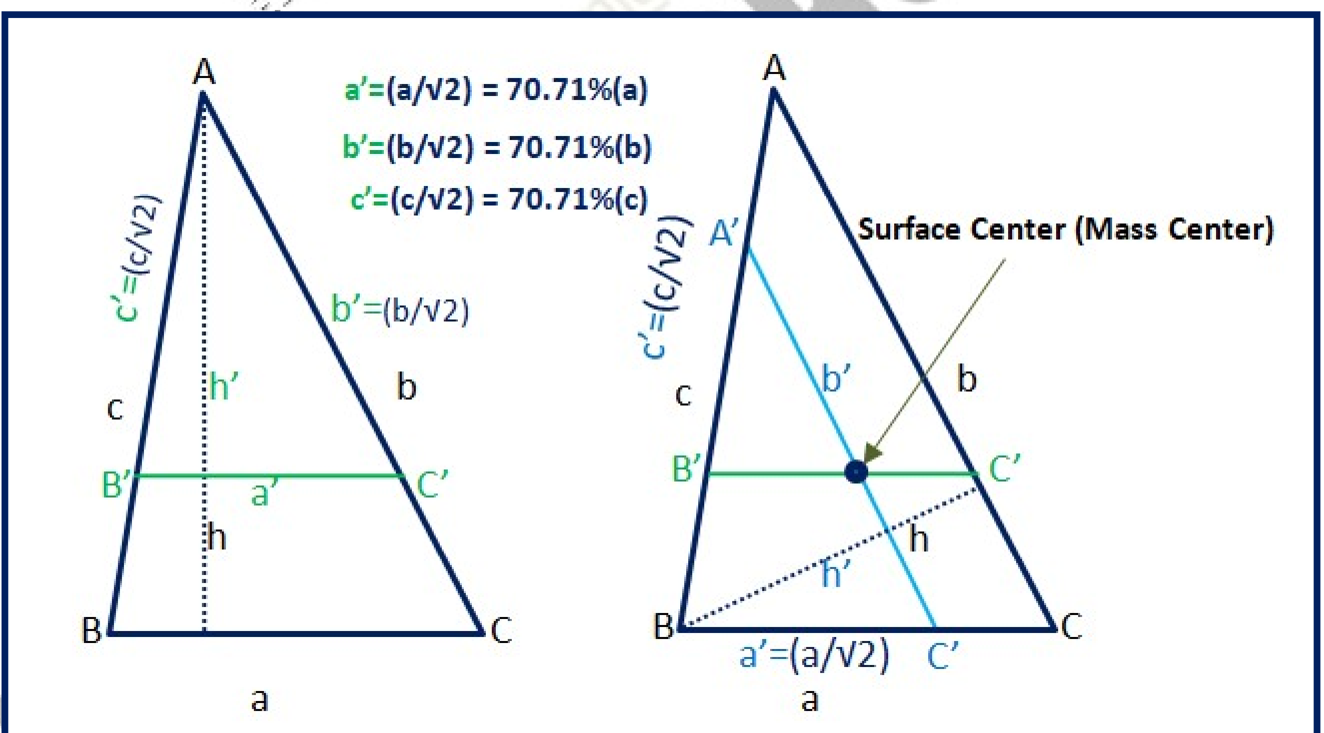

Figure-4.1: The Surface Center of Triangle

Parallel Transforming Percentage (PTP) Theorem

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Let A1 is the area of the triangle AB'C' with sides a', b', c' and h', A2 is the area of trapezoid B'C'BC of the first drawing. To find the mass center, A1 must be equal A2.

We have, $A1 = \frac{a'h'}{2}$; And $A2 = \frac{a+a'}{2}(h-h')$

➔ $a'h' = (a+a')(h-h') = ah - ah' + a'h - a'h'$

➔ $2a'h' + ah' = ah + a'h$

➔ $(2a' + a)h' = (a+a')h$

➔ $\frac{h'}{h} = \frac{a+a'}{2a'+a}$

Then apply PTP theorem, $K_r = \frac{h'}{h} = \frac{b'}{b} = \frac{a'}{a}$; → b' = b K_r **;** → a' = a K_r

➔ Let K_r = k; Then, $\frac{h'}{h} = \frac{a+a'}{2a'+a}$ → $k = \frac{a+a}{2ak+a}$ → $2ak^2 + ak = a + ak$

➔ $2ak^2 = a$

➔ Then the key ratio to have this surface center lying on line B'C' is,

➔ $K_r = k = \frac{1}{\sqrt{2}}$

Similar proof for other sides by applying PTP theorem to the 2nd drawing $K_r = \frac{1}{\sqrt{2}} = \frac{a'}{a} = \frac{b'}{b} = \frac{c'}{c}$; we will also find the key ratio of A'C' at 70.71% of AC to have the area of both triangles with equal value.

So, the surface center or mass center must be the point crossing by at least 2 lines in parallel with the bases in the parallel transforming percentage of $\frac{1}{\sqrt{2}}$ or 70.71% of the triangle side, $\boldsymbol{a' = \frac{a}{\sqrt{2}}}$; $\boldsymbol{b' = \frac{b}{\sqrt{2}}}$; $\boldsymbol{c' = \frac{c}{\sqrt{2}}}$

$$\boldsymbol{a' = \frac{a}{\sqrt{2}};\quad b' = \frac{b}{\sqrt{2}};\quad c' = \frac{c}{\sqrt{2}}}$$

Example:

Given, a= $12; b = 25; c = 23$; and h_a = 22.9129;

$A1 = \frac{a'h'}{2} = \left(\frac{12}{\sqrt{2}}\right)\left(\frac{22.9129}{\sqrt{2}}\right)\frac{1}{2} = 68.7387$;

$A2 = \frac{a+a'}{2}(h - h') = \left(\frac{12}{\sqrt{2}} + 12\right)\left(22.9129 - \frac{22.9129}{\sqrt{2}}\right)\frac{1}{2}$ = 68.7387

Where $A = \frac{ah}{2} = 12 * \frac{22.9129}{2} = 137.4774 = A1 + A2 = 2(68.7387)$

2. Find Surface Center of Trapezoid

Apply PTP theorem to find the surface center of trapezoid, the surface center is also known as the mass center of the same material with the same thickness and density.

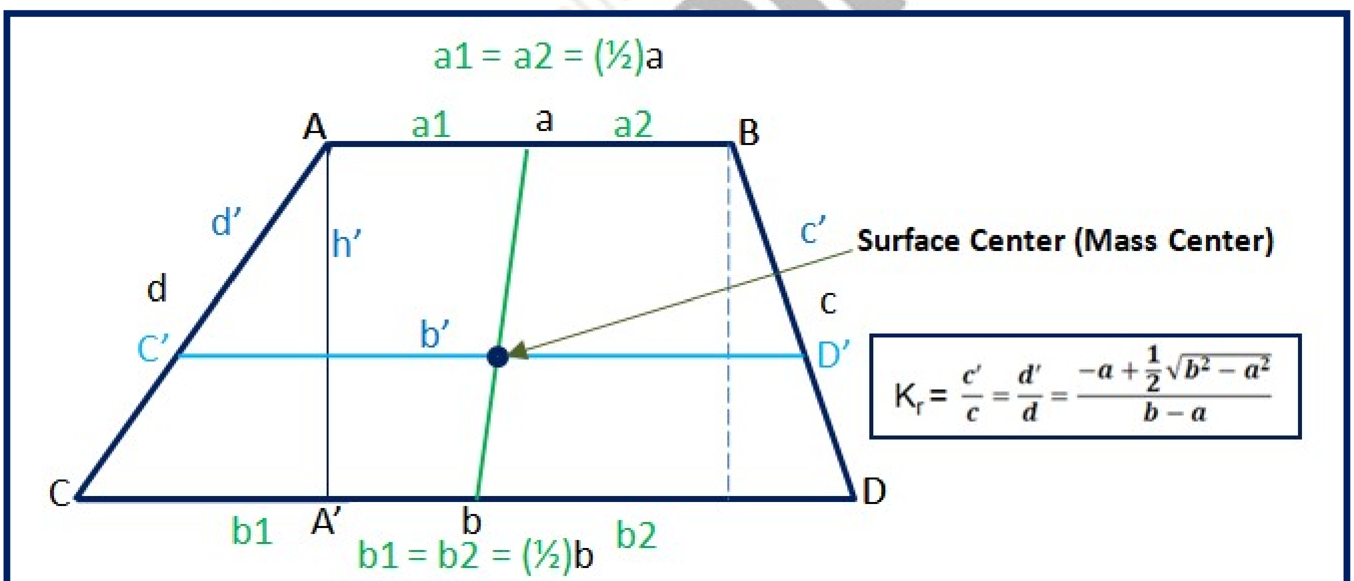

Figure-4.2: The Surface Center of Trapezoid

First, let find the 2 easy halves of the trapezoid by draw a line straight from the center of the base '**a**' and connect to the center of the base '**b**'. This line will create 2 trapezoids with the same area with the same high and the same base

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a1 = a2; and b1 = b2. Then we find another line which parallels with the bases that divide the trapezoid into 2 new trapezoids ABC'D' and C'D'CD in equal area. Let A1 is the area of the trapezoid ABC'D' with 2 bases a, b' and sides c', d' with the high h', A2 is the area of trapezoid C'B'CD with 2 bases b', b and sides (c-c'), (d-d') with the high (h-h'). To find the mass center, A1 must be equal A2 or A1 = A2 = ½ A; Where A is the area of the original trapezoid ABCD.

We have,

$$A1 = \frac{(a+b')h'}{2};\ A2 = \frac{(b+b')(h-h')}{2};\ And\ A = \frac{(a+b)h}{2}$$

→ $(a+b)h = 2(a+b')h'$

Let $K_r = k$; Then apply PTP theorem, $k = \frac{h'}{h} = \frac{b'}{b} = \frac{c'}{c} = \frac{d'}{d} = \frac{a+b}{2(a+b')}$

From PTP theorem we have, $b' = a + k(b-a)$

Then, substituted to the above equation we have,

$$k = \frac{a+b}{2(a+[a+k(b-a)])}$$

→ $(a+b) = 2k(2a+bk-ak)$

→ $(a+b) = 4ak + 2(b-a)k^2$

→ $2(b-a)k^2 + 4ak - (a+b) = 0;$

Apply quadrant equation to find **k** we have,

$$k = \frac{-4a + \sqrt{16a^2 - 4*2(b-a)(b+a)}}{2*2(b-a)} = \frac{-2a + \sqrt{4a^2 + 2(b-a)(b+a)}}{2(b-a)}$$

After simplifying we have k $= \frac{-a+\sqrt{\frac{1}{2}(a^2+b)}}{b-a}$

$$K_r = \frac{h'}{h} = \frac{c'}{c} = \frac{d'}{d} = \frac{-a+\sqrt{\frac{1}{2}(a^2+b^2)}}{b-a}$$

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Example:

Given the trapezoid with a = 15.5; b = 27; h =12; c = 14; and d = 12.743

We have $\frac{h'}{h} = \frac{-a+\sqrt{\frac{1}{2}(a^2+b^2)}}{b-a} = \frac{-1\ \ .5+\sqrt{\frac{1}{2}(15.5^2+27^2)}}{27-\ \ \ .5} = 0.5665 \quad or \ \ 56.65\%$

Now, we can confirm with this percentage. We have b' = 15.5 + 0.5665(27-15.5) = 22.01; h' = 0.5565(12) = 6.8;

$$A1 = \frac{(15.5 + 22.0142)6.8}{2} = 127.5$$

$$A2 = \frac{(22.0142 + 27)(12 - 6.7974)}{2} = 127.5$$

Where $A = \frac{(a+b)h}{2} = \frac{(15.5+27)12}{2} = 255 = A1 + A2 = \ 2(127.5)$

Irregular Shape Surface Center:

For any irregular shapes in general, we can use computer algorithm to find at least 2 lines that divide the shape into 2 sides with equal areas with the simplest shapes as possible. These 2 lines cross at a point, and this point is the surface center.

3. Find Object Center of Cone and Pyramid

Apply PTP theorem to find the object center of cone and pyramid, the **object center** is also known as the **mass center** of an object with the same material and density.

Parallel Transforming Percentage (PTP) Theorem

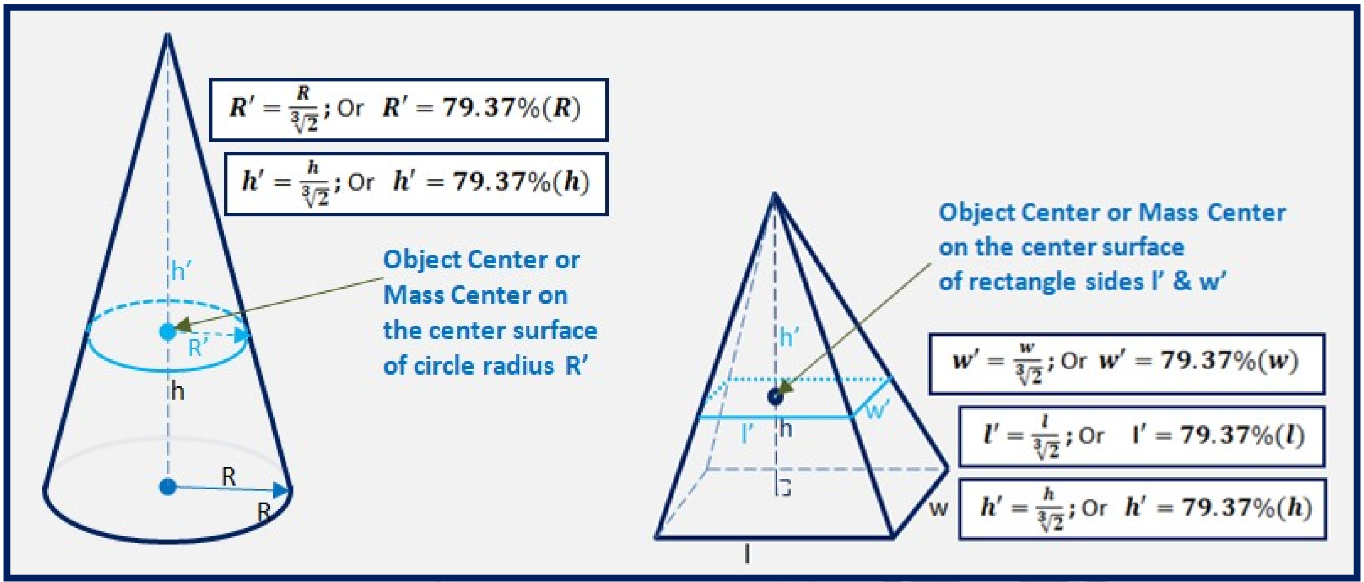

Figure-4.3: The Object Center of Cone & Pyramid

Let K_r = k; Then apply PTP theorem for triangle, $k = \frac{R'}{R} = \frac{h'}{h}$

$We\ have\ Cone\ volume\ formla: V = \frac{1}{3}\pi R^2 h$

So, $V' = \frac{1}{3}\pi R'^2 h'$; To find the object center of the cone, $V' = \frac{1}{2}V$

Then, d $V' = \frac{1}{3}\pi R'^2 h' = \frac{1}{2}\frac{1}{3}\pi R^2 h$ → $2\pi R'^2 h' = \pi R^2 h$ → $\frac{R'^2 h'}{R^2 h} = \frac{1}{2}$;

→ $We\ have, k = \frac{R'}{R} = \frac{h'}{h}$ → Then $\frac{R'^2 h'}{R^2 h} = \frac{R'^3}{R^3} = \frac{h'^3}{h^3} = \frac{1}{2}$

Finally, we have the K_r = $k = \frac{R'}{R} = \frac{h'}{h} = \frac{1}{\sqrt[3]{2}}$

We can find the ½ volume of the cone for the high and radius as following,

$\boldsymbol{h' = \frac{h}{\sqrt[3]{2}}}$; Or $\boldsymbol{h' = 79.37\%(h)}$

$\boldsymbol{R' = \frac{R}{\sqrt[3]{2}}}$; Or $\boldsymbol{R' = 79.37\%(R)}$

For pyramid, apply PTP theorem for triangle, Let K_r = $k = \frac{h'}{h} = \frac{w'}{w} = \frac{l'}{l'}$;

$$We\ have\ the\ pyramid\ volume\ formula\ \ V = \frac{1}{3}wlh$$

So, $V' = \frac{1}{3}w'l'h'$;

We can prove the same as the cone above, to find object center,

$V' = \frac{1}{2}V = \frac{1}{2}\frac{1}{3}\ wlh = \frac{1}{3}w'l'h'$; We have, $k = \frac{h'}{h} = \frac{w'}{w} = \frac{l'}{l}$

$Then\ we\ have,\ \frac{1}{3}(w'l'h') = \frac{1}{2}\frac{1}{3}\ wlh \ \rightarrow \frac{(w'l'h')}{\text{wlh}} = \frac{1}{2}$

$$Where, \frac{w'}{w} = \frac{l'}{l} = \frac{h'}{h}; Then\ from\ the\ above\ we\ have, \frac{w'^3}{w^3} = \frac{l'^3}{l^3} = \frac{h'^3}{h^3} = \frac{1}{2}$$

Finally, we have the K_r = $k = \frac{h'}{h} = \frac{w'}{w} = \frac{l'}{l} = \frac{1}{\sqrt[3]{2}}$;

We can find the ½ volume of the pyramid for the high, width and length as following,

$\boldsymbol{h' = \frac{h}{\sqrt[3]{2}}}$; Or $\boldsymbol{h' = 79.37\%(h)}$

$\boldsymbol{w' = \frac{w}{\sqrt[3]{2}}}$; Or $\boldsymbol{w' = 79.37\%(w)}$

$\boldsymbol{l' = \frac{l}{\sqrt[3]{2}}}$; Or $\boldsymbol{l' = 79.37\%(l)}$

The parallel transforming percentage shows the same as the cone above.

__Example__:

Given the cone with R = 12; h =19;

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Then, R' = 0.7937 * 12 = 9.5244; and h' = 0.7937 * 19 = 15.0803;

$$V' = \frac{1}{3}\pi R'^2 h' = \frac{1}{3}\pi(9.5244^2)(15.0803) = 1432.5653$$

$V = \frac{1}{3}\pi R^2 h = \frac{1}{3}\pi(12^2)(19) = 2865.1325;$

V = 2865.1325 which is equal to 2(1432.5653); So, the object center is right at the center surface of the circle plane with radius R' = 9.5244.

Given the pyramid with h = 15; l = 9; w = 7;

Then,

h' = 0.7937 * 15 = 11.9055; l' = 0.7937 * 9 = 7.1433; w' = 0.7937 * 7 = 5.5559;

$$V' = \frac{l'w'h'}{3} = \frac{(7.1433 * 5.5559 * 11.9055)}{3} = 157.5$$

$$V = \frac{lwh}{3} = \frac{(9 * 7 * 15)}{3} = 315$$

V = 315 which is equal to 2(157.5); So, the object center is right at center of the rectangle with sides l'=7.1433, w'=5.5559, at the high h' = 11.9055.

4. PTP Theorem triggers to other use cases and applications in real life

a. Paint color mixer; however the current color codes which have been used in computer for decades may need to revise to have a better color relative spectrum with the relative based color code for each color. When the color codes are correct relatively, then we can use computer for color mixer to predict the outcome color better and more accurate after mixing with many different colors with different percentage of color paint.

b. Weight and volume measurement and calculations.

c. Estimate object distance with camera or binocular.
d. Gear and trolley mechanism calculations.
e. Help to build giant airplane based on a well testable dimension.

V. Summary

The Parallel Transforming Percentage (PTP) theorem can be applied widely to calculate lines, surfaces and volumes for any shapes and objects relatively to its original one. The PTP theorem is applied in this document to find surface center and object center and to solve the mystery of ellipse which currently has incorrect definition of two focuses. We now can have short definition of ellipse, ellipse is a circle which rotates an angle on its plane.

Beside the PTP theorem, I also proved other theorems to support the PTP theorem. One of these theorems solves the common misunderstanding of surface area verse circumference ratio. Circle is the magic shape which has infinite sides, and which ever shape that has ‘**n**’ sides equal (e-chord lines in its circle) always has larger area to circumference ratio compare to the shapes with the same number of ‘**n**’ sides not equal. This is also true for objects or 3D shapes with ‘**n**’ sides equal have largest volume than other objects with ‘**n**’ sides not equal; and sphere is the magic object with infinite sides, has largest volume to surface area ratio compare to other objects.

This PTP theorem is not just for geometry, this PTP theorem also triggers for many great ideas from weight to density percentage, color mixer in percentage to create giant airplane start from a testable dimension. Based on the greatest size of an object, the PTP theorem will be applied as <u>the numbers within 0 and 1</u>, and 1 is 100%. And the measurement is based on percentage without using actual number and unit for easier and better calculating and transforming of multi-directional transforming. The multi-directional transforming will be more common and useful in the future to help computer simulations faster and easier.

Parallel Transforming Percentage (PTP) Theorem

Biography

About myself, I am Henry Viet Pham, and I am a father of 3 sons, Alexander Le Pham, Andrew Le Pham, and Harry Quoc Pham and my wife Celine Nguyet Tran. I was born in Vietnam in 1972/08/23, and I came to United States in 1991 as a military and political immigrant with my father and family members. My father Nu Pham who served as a Lieutenant in military during Vietnam War in 1975, and my mother is Thong Thi Tran with my sisters are Nguyet Thi Pham, Jessie Nga Pham and Tiffany Tuyen Pham, and my brothers are Duc Hong Pham, Kevin Tri Pham, Danny Phuc Pham, and Andy Quy Pham.

About Education, I came to United States after finished my high school in Vietnam, and I continued my education right after came to U.S. and got my Bachelor Degree in Electrical and Computer Engineering at Calpoly Pomona, California in 1998. I am interested in Engineering and Science, and I have done many researches and self-study since I graduated in 1998 and continue researching and inventing with total of 9 inventions which have been submitted for patents from June 2021 to October 2023, and I still have many more inventions to work on and open the Cloud OS Company for business.

About my works and inventions, I am a sole inventor of a total of 9 inventions which have been submitted from June 2021 to October 2023 as followings.

Parallel Transforming Percentage (PTP) Theorem

1. Invention Title: New Way to protect WiFi Network from Hackers – with U.S. Patent PCT No.: 29/788,607; Submitted: 2021/07/01;

2. Invention Title: THE G-CODE – with U.S. Patent PCT No.: 29/806,573 => PCT/US22/70704; and International Patent: PCT/IB2022/000112; Submitted: 2021/09/03;

3. Invention Title: The Cloud OS - Operating System – with U.S. Patent PCT No.: PCT/US21/71689; and International Patent: PCT/IB2021/000683; Submitted: 2021/10/02;

4. Invention Title: The LPS - Local Positioning System – with U.S. Patent PCT No.: PCT/US21/72562; and International Patent: PCT/IB2021/000949; Submitted: 2021/11/23;

5. Invention Title: Greatest Performance Hard Drive (G-Drive) – with U.S. Patent PCT No.: PCT/US21/72563; and International Patent: PCT/IB2021/000961; Submitted: 2021/11/23;

6. Invention Title: Cell eMap Live Updates System – with U.S. Patent PCT No.: PCT/US22/79368; and International Patent: PCT/IB2022/000685; Submitted on 2022/11/07;

7. Invention Title: LPS Navigation System – with U.S. Patent PCT No.: PCT/US22/79369; and International Patent: PCT/IB2022/000671; Submitted on 2022/11/07;

8. Invention Title: Emergency Traffic Lights Routing System – with U.S. Patent PCT No.: PCT/US22/82343; and International Patent: PCT/IB2022/000791; Submitted on 2022/12/23;

9. Invention Title: G-ROUTING ALGORITHM METHODOLOGY -- with U.S. Patent PCT No.: PCT/US22/82347; and International Patent: PCT/IB2022/000800; Submitted on 2022/12/23;

About my business, The Cloud OS Company (website: www.TheCloudOSCenter.com) business uses mainly Invention #3: The Cloud OS – Operating System, Invention #5: Greatest Performance Hard Drive (G-Drive), and Invention #9: G-ROUTING ALGORITHM METHODOLOGY. The Cloud OS Company business brings the world to the next level of World Computing Infrastructure Modern with the main purposes to secure users' data and secure entire computer networking around the world or the World eWeb with the new technology of Neighbor-to-neighbor checking methodology and Neighbor-to-neighbor routing technology, and applying the new dynamic protocol technology for data transferring with the high secure of the 4K Number Encryption.